V/V_m method for small planet microlensing events

Udalski, A. et al. 2018, ACTA ASTRONOMICA, 68: 1-42.

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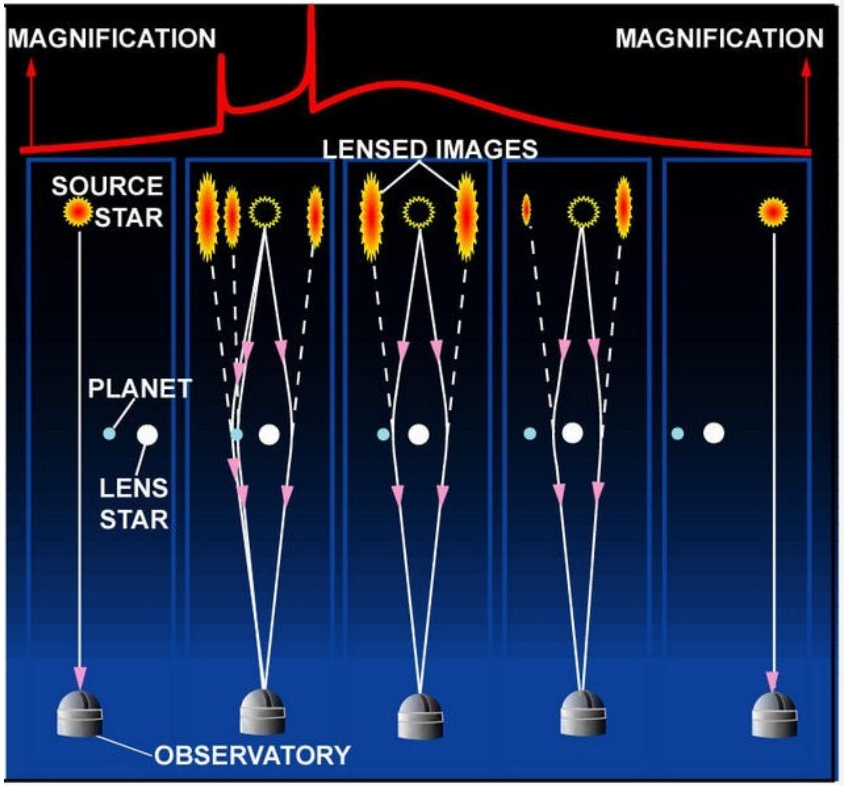
Introduction

Microlensing events with $q < 10^{-4}$

(1) OGLE-2005-BLG-169
(2) OGLE-2005-BLG-390
(3) OGLE-2007-BLG-368
(4) MOA-2009-BLG-266
(5) OGLE-2013-BLG-0341
(6) OGLE-2016-BLG-1195
(7) OGLE-2017-BLG-0173
(8) OGLE-2017-BLG-1434



Retrieved from OGLE web

→ Detectability of $q < 10^{-4}$ events?
→ Mass function of $q < 10^{-4}$ lenses?

V/V_m method

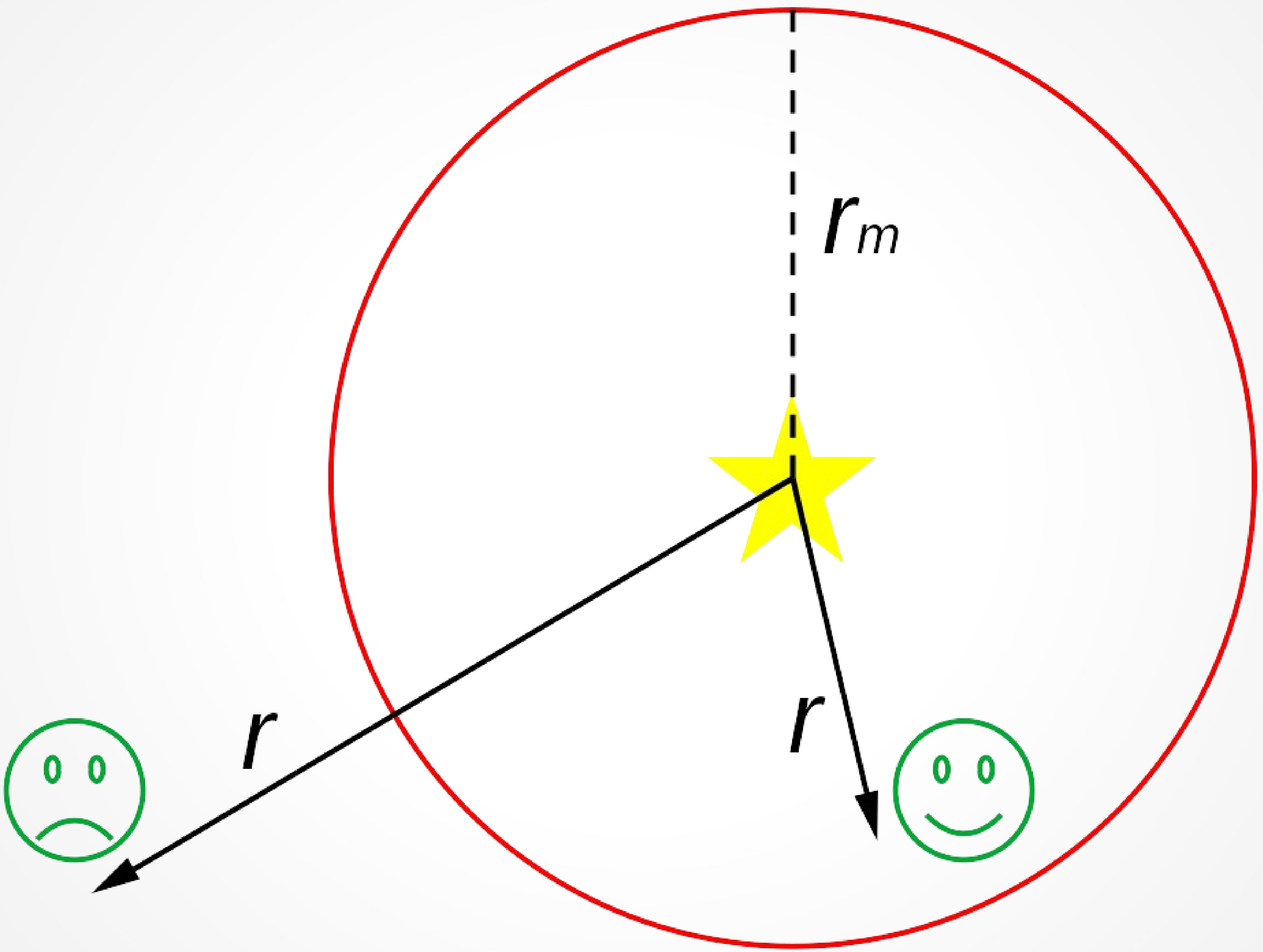
V/V_m method

- Kafka (1967)
- Schmidt (1968)
- Lynden-Bell (1971)

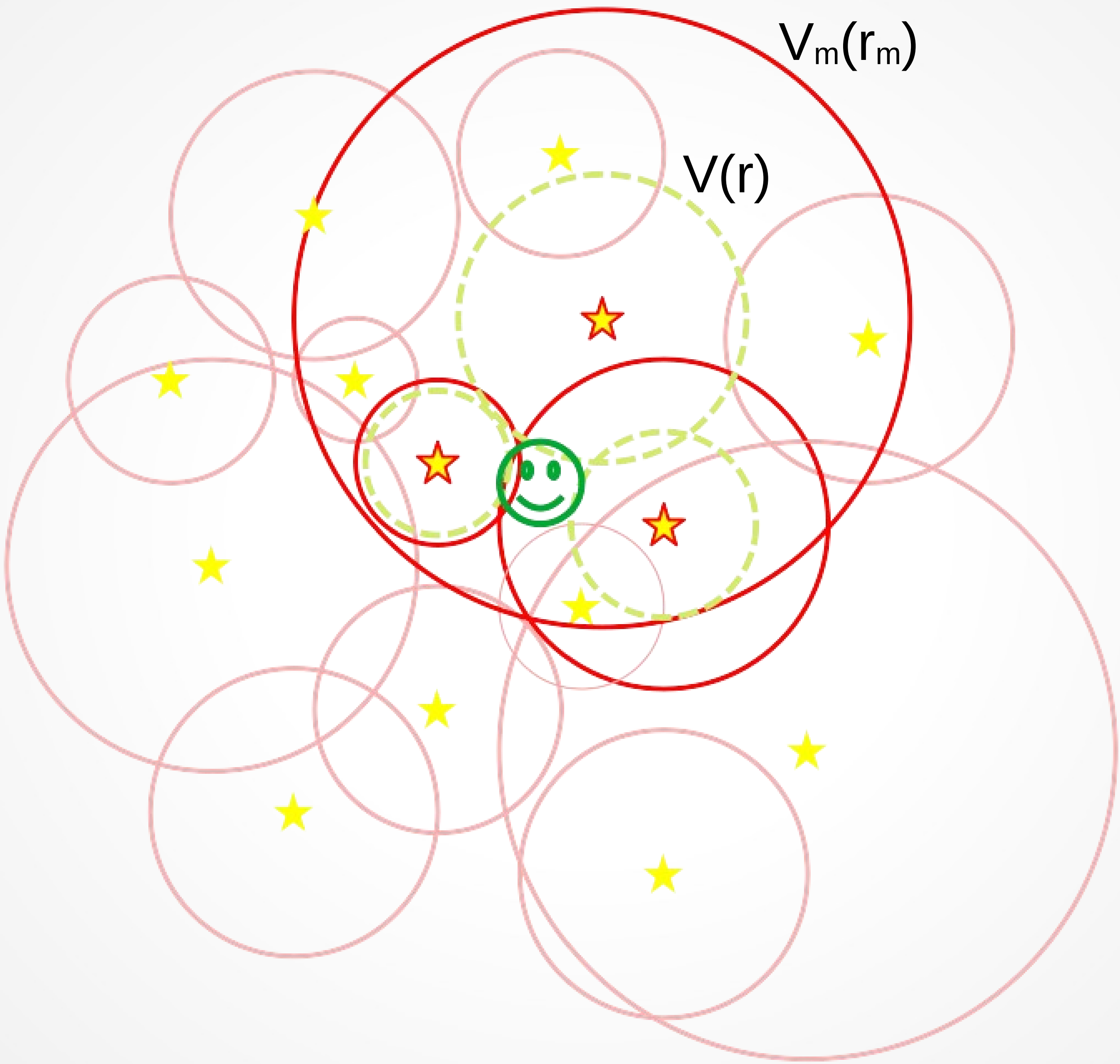
Ratio for the distribution of “detectable” samples as a probability

$$\frac{V}{V_m} = \frac{\text{Integrated flux about the distance from the source to detector } (r)}{\text{Integrated flux about the detectable distance limit } (r_m)}$$

V/V_m method



V/V_m method



V/V_m method

Acceptable samples : $r < r_m$

$$\langle V/V_m \rangle = \frac{\int_{L_{min}}^{L_{max}} dL\, \phi(L) \int_0^{r_m(L)} (V/V_m)\, 4\pi r^2 \rho_0\, dr}{\int_{L_{min}}^{L_{max}} dL\, \phi(L) \int_0^{r_m(L)} 4\pi r^2 \rho_0\, dr}$$

L = Luminosity
$\phi(L)$ = Luminosity function
ρ_0 = uniform density

For equally distributed samples :

$$\langle V/V_m \rangle \rightarrow \frac{1}{2} \qquad \sigma_{\langle V/V_m \rangle} \rightarrow \frac{1}{\sqrt{12N}}$$

N = size of samples

V/V_m for microlensing

Udalski et al. (2018) :

$$r_i = \frac{\int_{q_i}^{q_{max}} dlnq' F(q') P_i(q')}{\int_{0}^{q_{max}} dlnq' F(q') P_i(q')}$$

q = Lens mass ratio
F(q) = Mass ratio function
P = Planet confirmation probability ($q' \neq q_i$)

$$r_i \rightarrow \frac{\int_{q_i}^{q_{max}} dlnq' F(q')}{\int_{q_{min,i}}^{q_{max}} dlnq' F(q')}$$

For $P_i(q) = 0$ or 1

V/V_m for microlensing

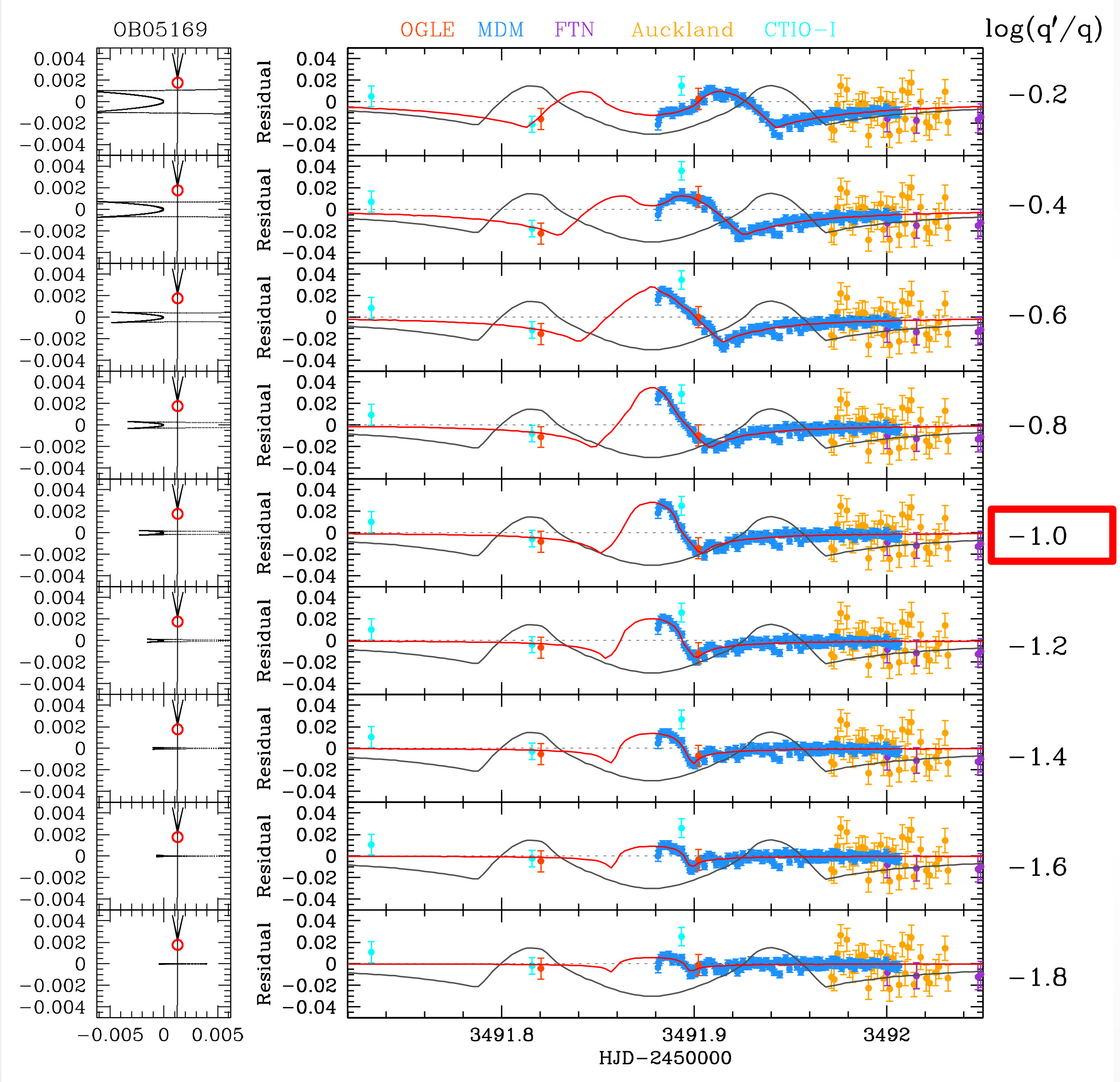
Criteria for finding q_{min} :

- $\log q < -4$
- $\sigma(\log q) < 0.15$
- No alternate solutions with $\Delta\chi^2 < 10$ and $\Delta \log q > 0.3$

Microlensing events with $\log q < -4$:

(1) OGLE-2005-BLG-169
(2) OGLE-2005-BLG-390
(3) OGLE-2007-BLG-368
(4) MOA-2009-BLG-266
(5) OGLE-2013-BLG-0341
(6) OGLE-2016-BLG-1195
(7) OGLE-2017-BLG-0173
(8) OGLE-2017-BLG-1434

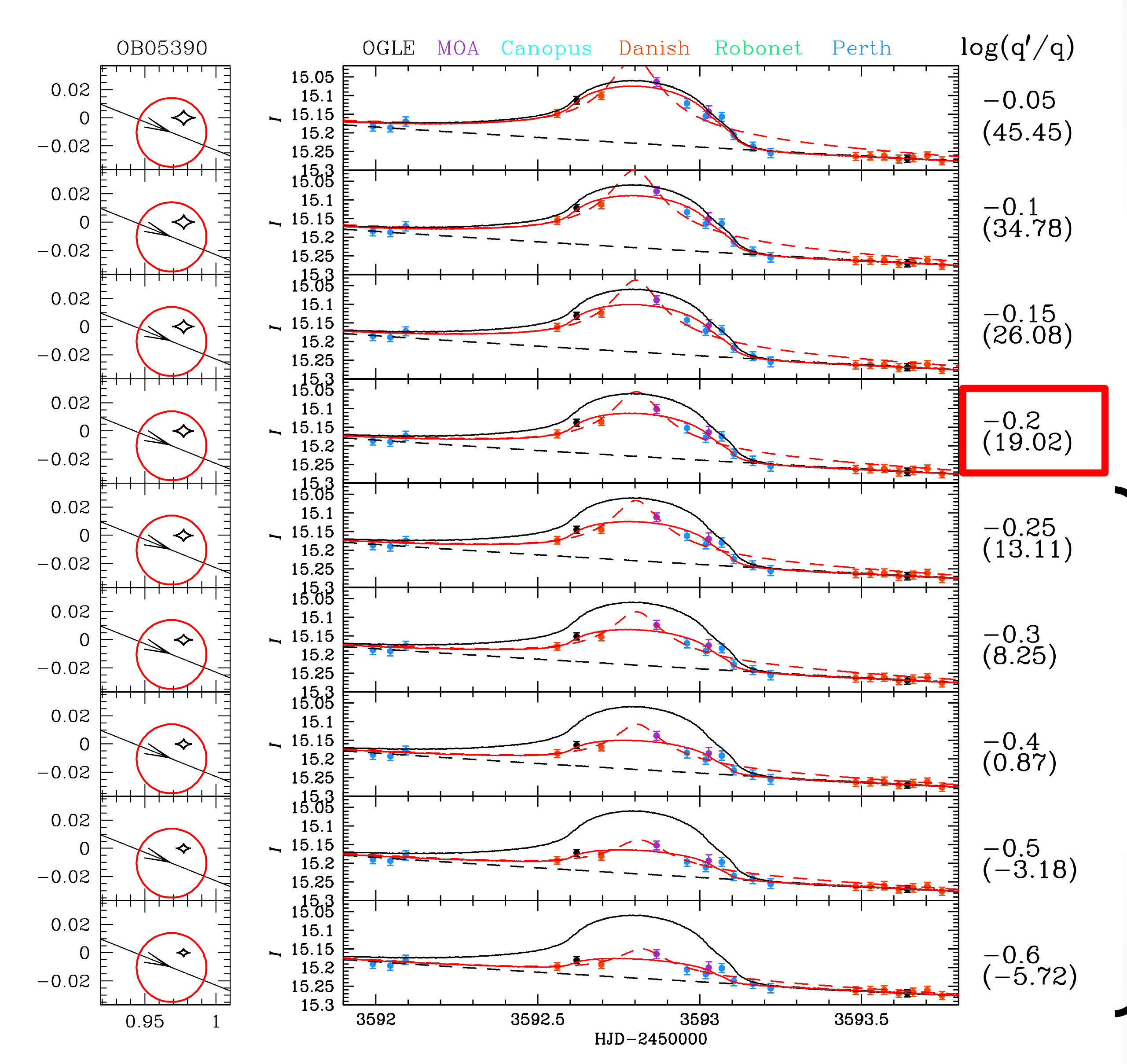
(1) OGLE-2005-BLG-169



$q = 6.1 \times 10^{-5}$

→ 0.3 mag offset for OGLE alert

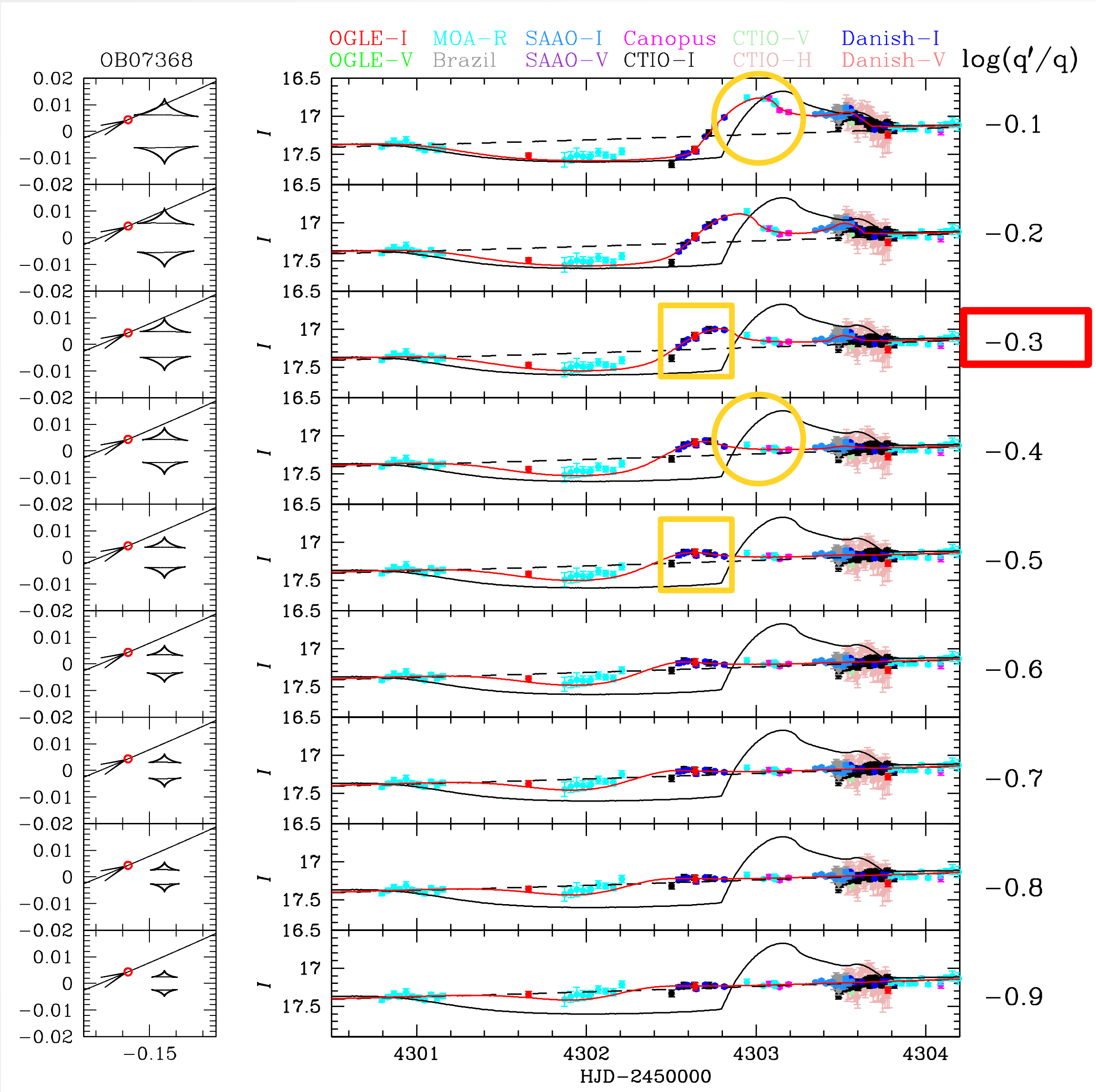
(2) OGLE-2005-BLG-390



$q = 7.6 \times 10^{-5}$

2L1S? 1L2S?

(3) OGLE-2007-BLG-368

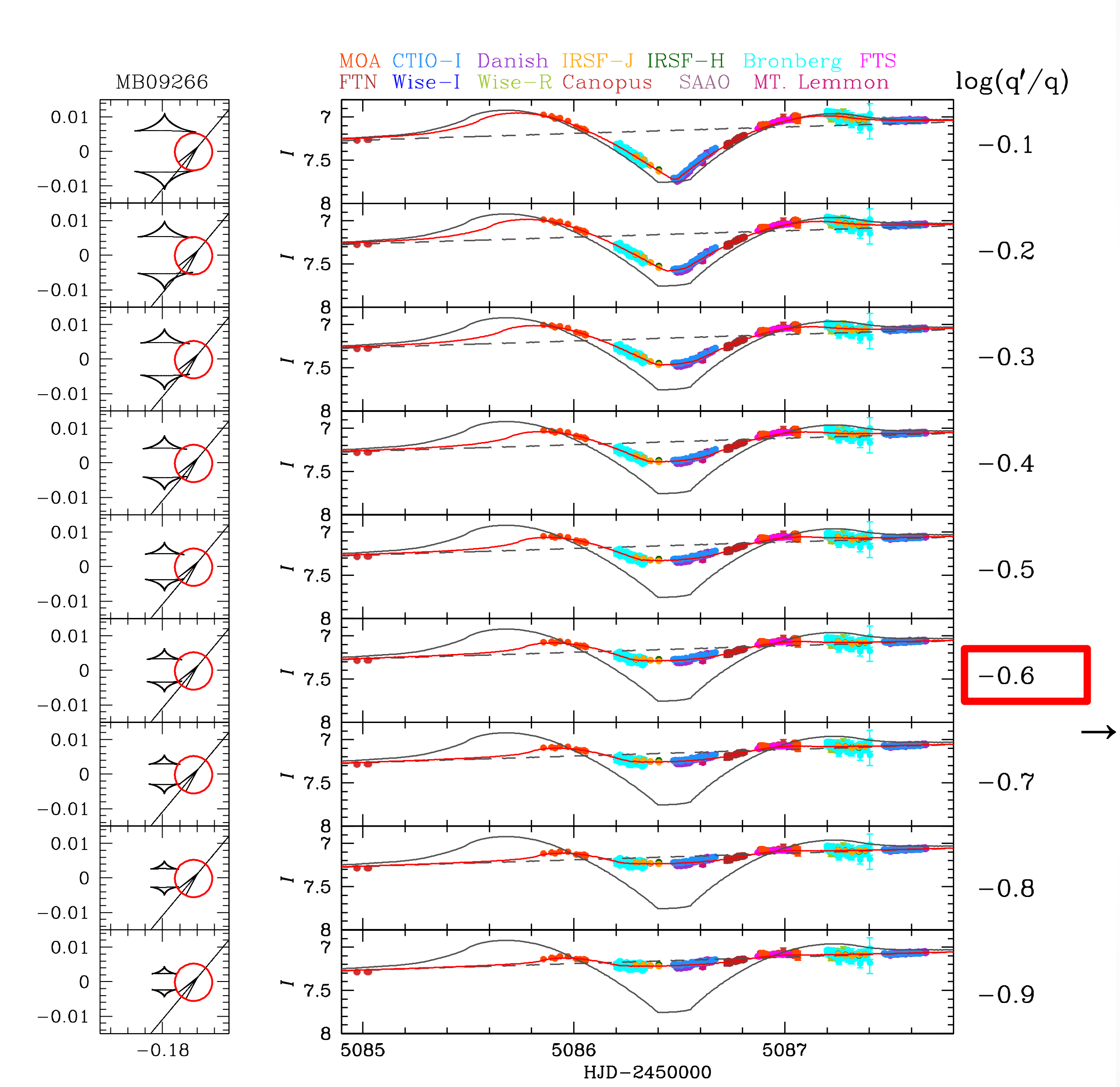


$q = 9.6 \times 10^{-5}$

→ $\Delta\chi^2 < 10, \Delta \log q > 0.3$

If no follow-ups …

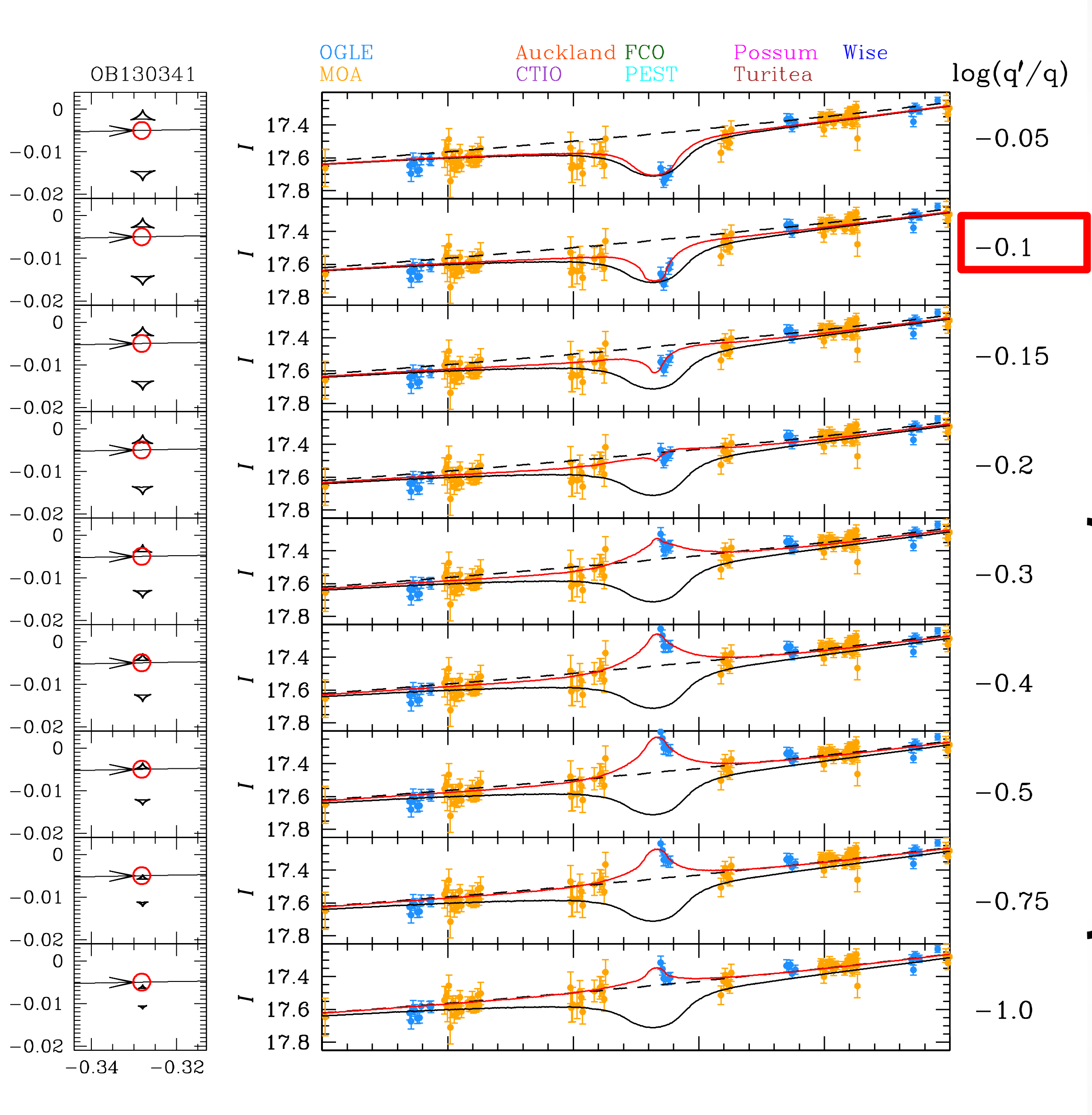
(4) MOA-2009-BLG-266



$q=5.6\times10^{-5}$

→ 0.1 mag offset for MOA alert

(5) OGLE-2013-BLG-0341

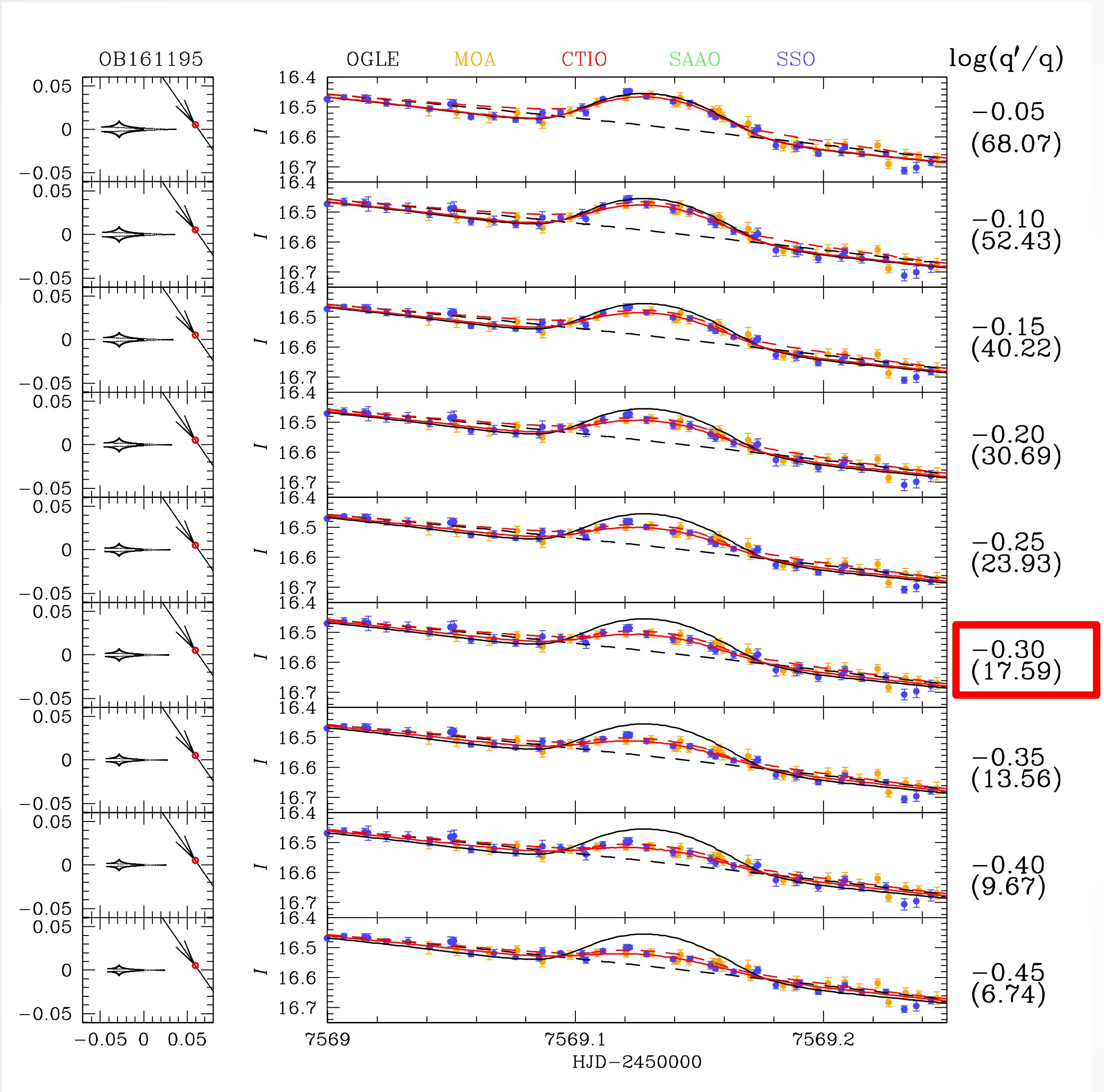


$q = 4.6 \times 10^{-5}$

Anomaly is clear, but ...
2L1S? 1L2S?

$\Delta \chi^2 \leqslant 66$

(6) OGLE-2016-BLG-1195

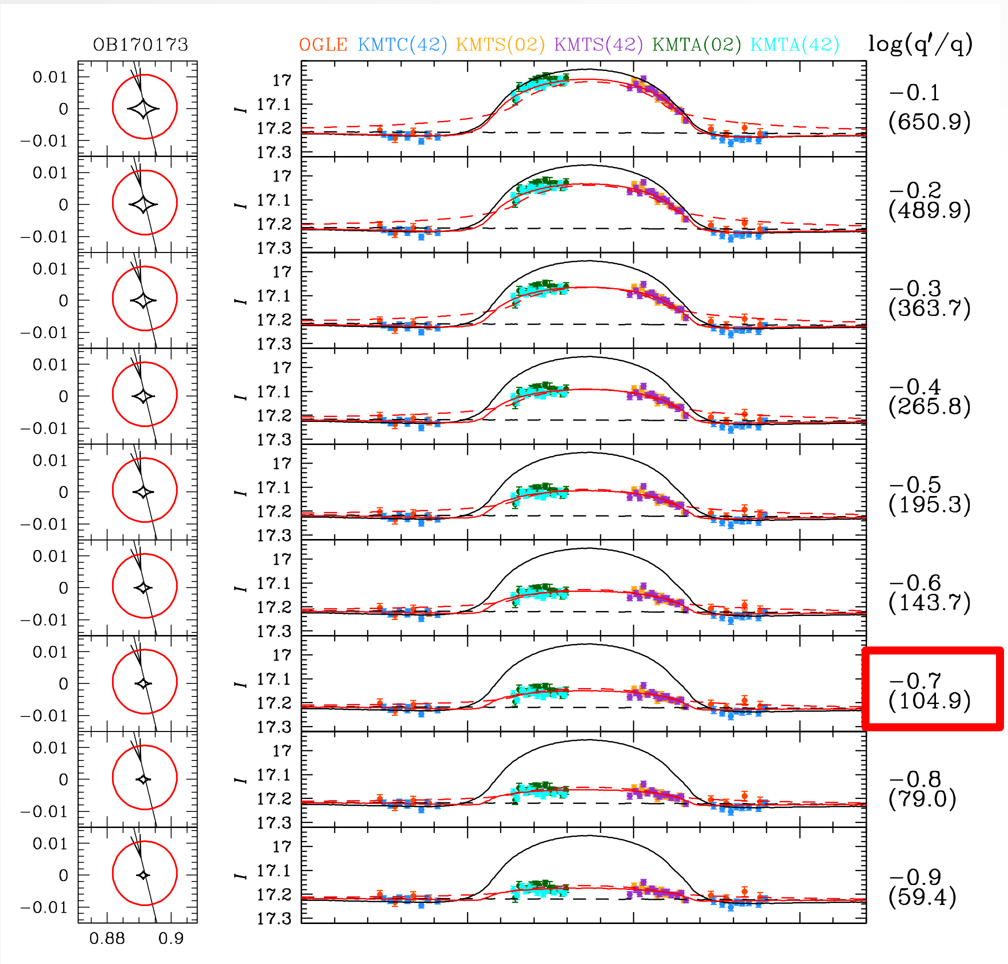


$q = 4.8 \times 10^{-5}$

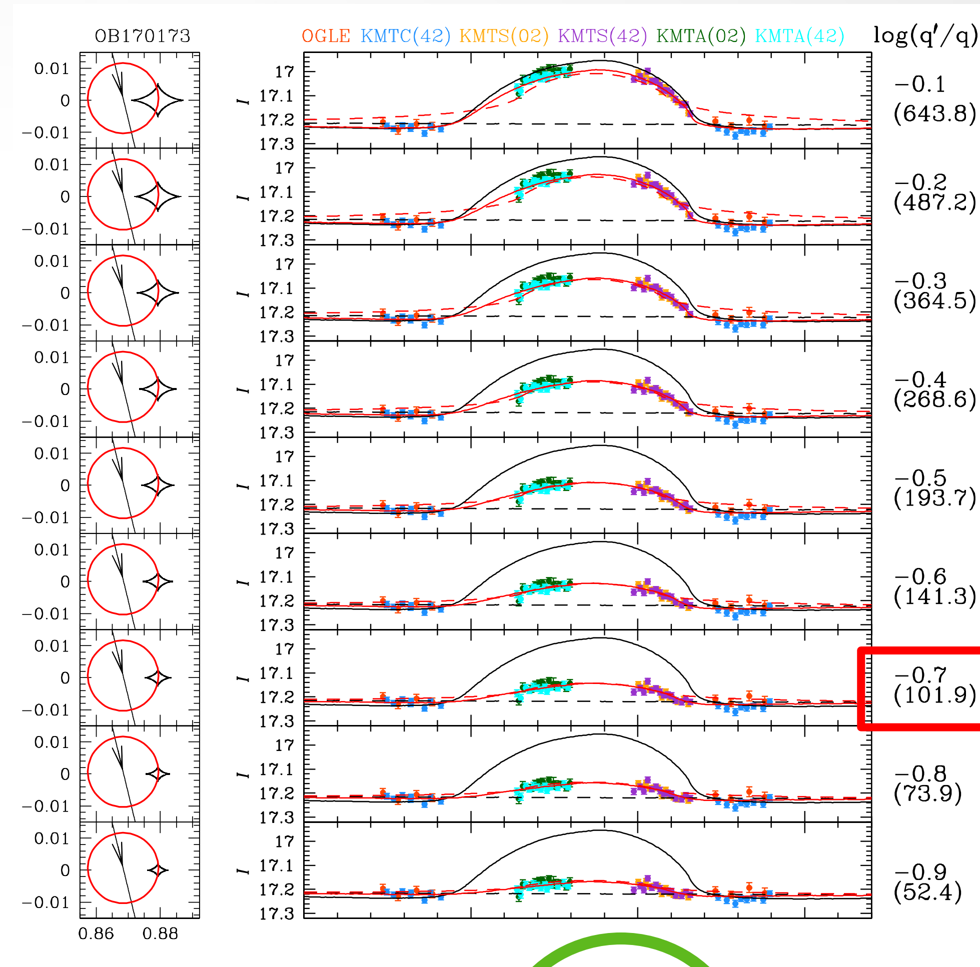
2L1S? 1L2S?

(7) OGLE-2017-BLG-0173

(A) $q=6.5\times10^{-5}$

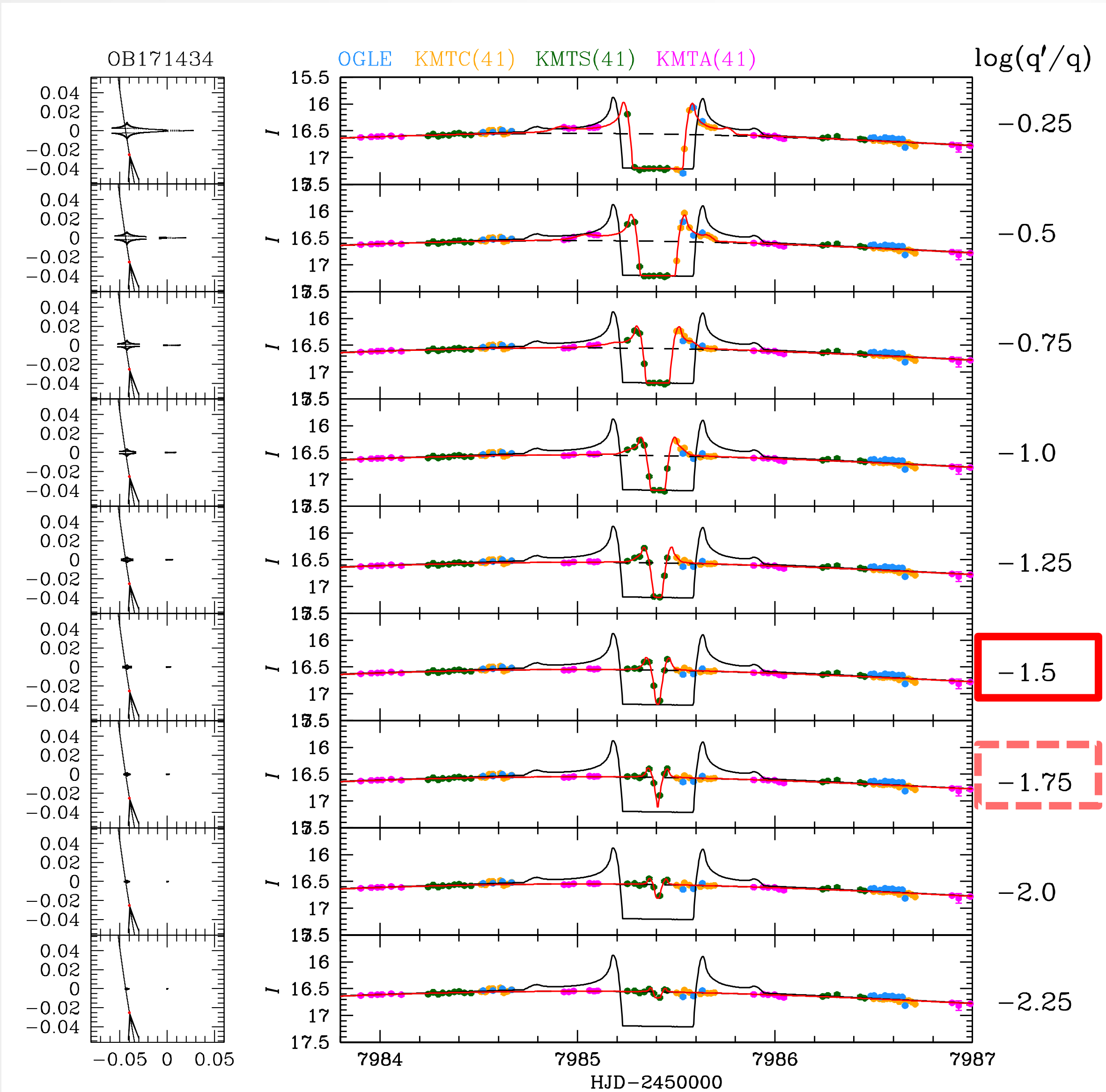


(B) $q=2.5\times10^{-5}$



For all $\log(q'/q)$ Between (A) and (B) $\rightarrow \Delta\chi^2 < 10, \Delta\log q > 0.3$

(8) OGLE-2017-BLG-1434

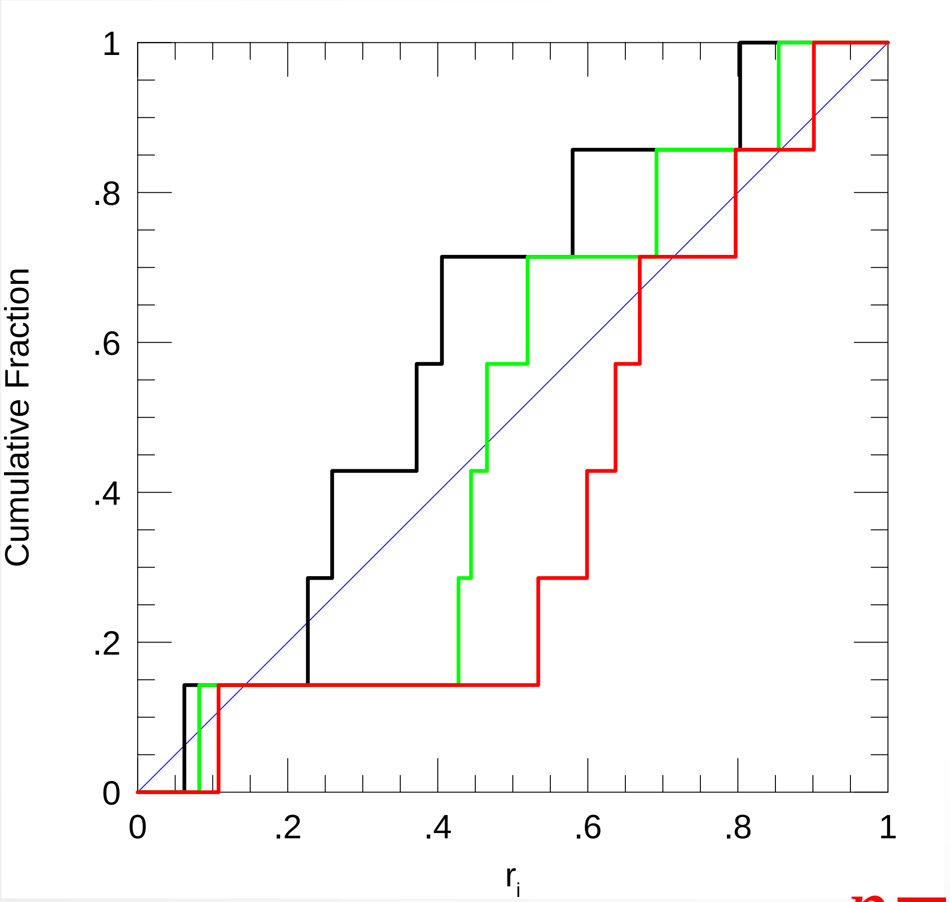


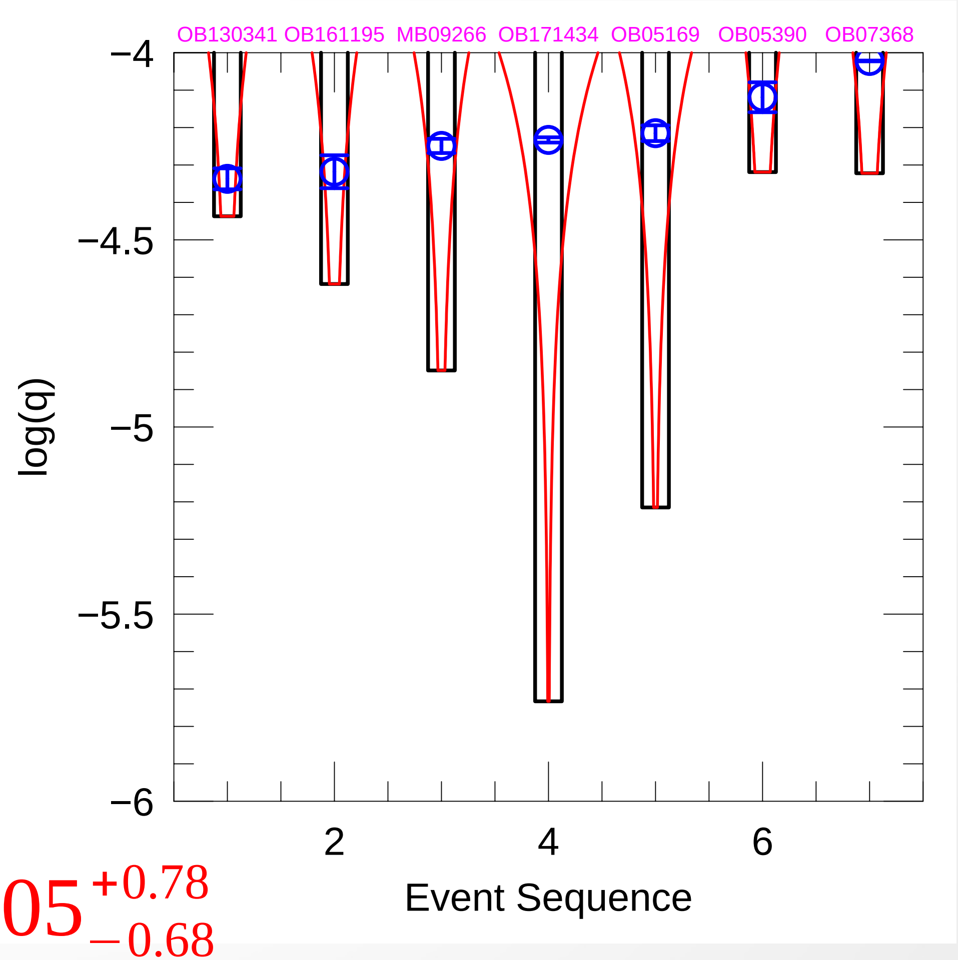
$q = 5.8 \times 10^{-5}$

→ Thanks KMTNet

Power law of the mass function

$$r_i = \frac{\int_{q_i}^{q_{max}} dlnq' F(q')}{\int_{q_{min,i}}^{q_{max}} dlnq' F(q')}$$ where $$F(q) = \frac{dN}{dlnq} \propto q^p$$, and assuming $$\langle r_i \rangle = \frac{1}{2} \pm \frac{1}{\sqrt{12N}}$$,





$$p = 1.05^{+0.78}_{-0.68}$$

Final solution

$\log_{10} q$	p	reference
> -1.4	0.32 ± 0.38	Shvartzvald et al. (2016)
-4.9 ~ -1.4	-0.50 ± 0.17	
≥ -3.75	-0.93 ± 0.13	Suzuki et al. (2016)
< -3.75	$0.6^{+0.5}_{-0.4}$	
< -4	$1.05^{+0.75}_{-0.68}$	Udalski et al. (2018)

$p = 0.73^{+0.42}_{-0.34}$ for $\log_{10} q < -4$

Summary

V/V_m method is a probability assumption

- Integrating effective parameter distribution (V)
- Taking the volume ratio to the maximum potential (V/V_m)
- Ideal mean ratio reaches to : mean=1/2, sigma=1/√(12N)

Lens mass ratio function $F(q) \propto q^p$ by V/V_m method

- q_{min} as a least requirement to confirm a planet
- Assuming the mean volume ratio about q → $\langle r_i \rangle = 1/2$
- $p \sim 0.73$ for $q < 10^{-4}$

Reference

- Schmidt, M., 1968, *Apj*, 151, 393.
- Shvartzvald, et al., 2016, *MNRAS*, 457, 4089.
- Suzuki, D., et al. 2016 *Apj*, 833, 145.
- Udalski, A., et al. 2018, *ACTA ASTRONOMICA*, 68, 1.