Hawkins et al. 2017 “Red clump stars and Gaia: calibration of the standard candle using a hierarchical probabilistic model”

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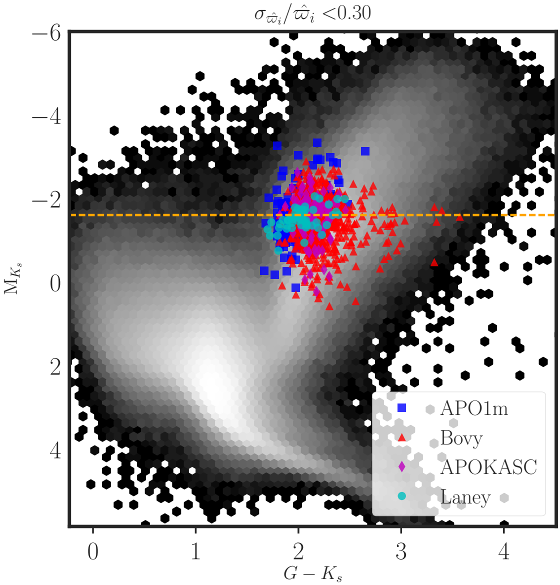
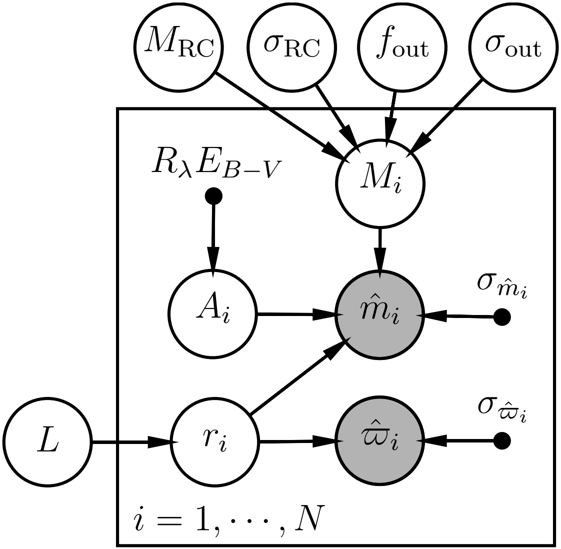
8.11.2022

Red clump stars

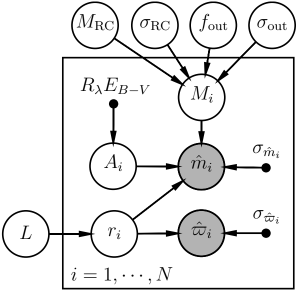


Probabilistic graphical model

- Shaded circles – observed data,
- Open circles – model parameters,
- Small filled circles – fixed parameters.



Distances



r_i – distance to star i
L – scale-length of the distance prior

$p(r_i \mid L) = \frac{1}{2L^3} r_i^2 \exp(-r_i/L)$
Bailer Jones (2015)

$\hat{\varpi}_i$ – measured parallax of star i
$\sigma_{\hat{\varpi}_i}$ – measured parallax uncertainty of star i

$p(\hat{\varpi}_i \mid r_i, \sigma_{\hat{\varpi}_i}) = \mathcal{N}(\hat{\varpi}_i \mid 1/r_i, \sigma_{\hat{\varpi}_i})$
$\mathcal{N}(x \mid x_{\rm mean}, \sigma_x)$ – normal distribution

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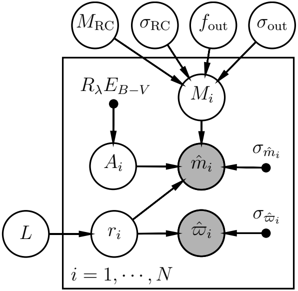

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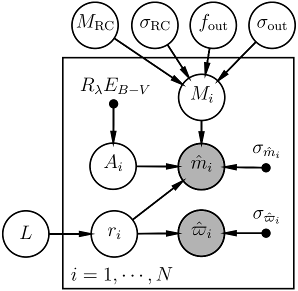

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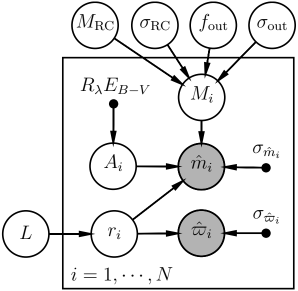

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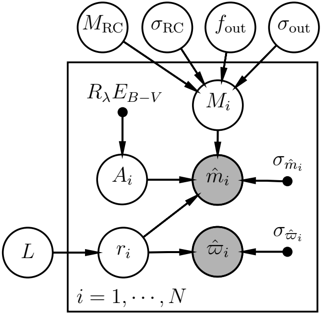
Predicted mean magnitude



M_i – predicted mean magnitude of star i

$M_{\rm RC}$ – mean magnitude of RC
$\sigma_{M_{\rm RC}}$ – dispersion of RC magnitudes
$\mathcal{N}(M_i \mid M_{\rm RC}, \sigma_{M_{\rm RC}})$ – normal distribution

$\sigma_{M_{\rm out}}$ – dispersion of magnitudes of outlier population
$f_{\rm out}$ – contamination fraction

$\theta_{\rm RC} = \{M_{\rm RC}, \sigma_{M_{\rm RC}}, \sigma_{M_{out}}, f_{\rm out}\}$

$$p(M_i \,|\theta_{\rm RC}) = (1 - f_{\rm out})\mathcal{N}(M_i \,|M_{\rm RC}, \sigma_{M_{\rm RC}}) + f_{\rm out}\mathcal{N}(M_i \,|M_{\rm RC}, \sigma_{M_{\rm out}})$$

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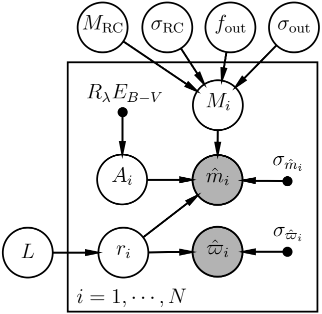

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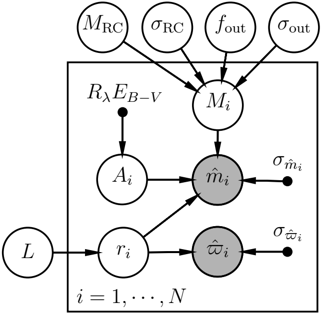

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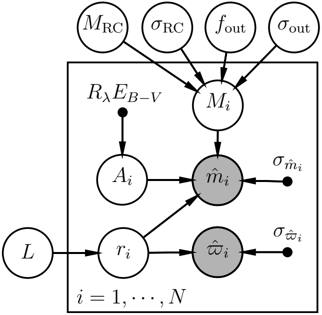

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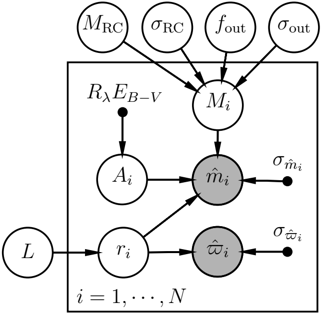

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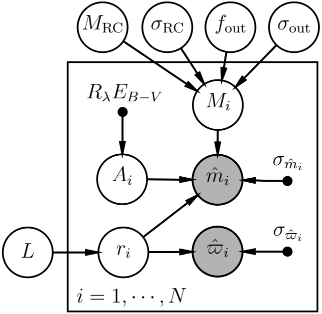
Extinction and observed brightness



$E(B-V)_i$ – reddening for star i from Green et al. (2015)
A_i – expected extinction
R_λ – extinction coefficient

$$A_i = R_\lambda \times E(B-V)_i$$

m_i – expected brightness of star i

$$m_i = M_i + 5\log_{10}(r_i) - 5 + A_i$$

$\hat{m}_i$ – measured brightness of star i
$\sigma_{\hat{m}_i}$ – measured brightness uncertainty of star i

$$p(\hat{m}_i|\ \theta_{\rm RC}, L, r_i, \sigma_{\hat{m}_i}) = \mathcal{N}(\hat{m}_i|m_i, \sigma_{\hat{m}_i})$$

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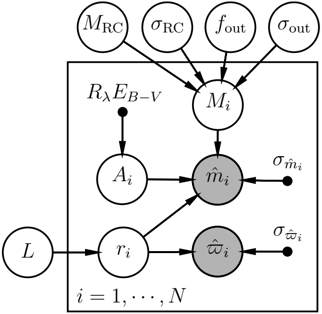

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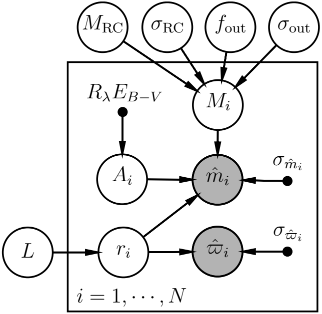

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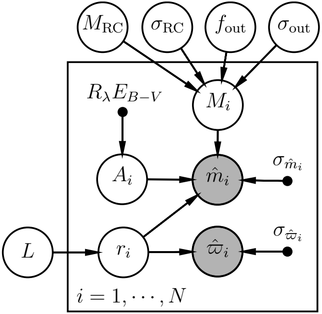

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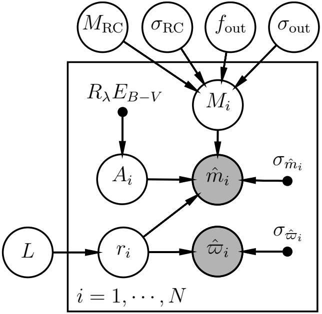

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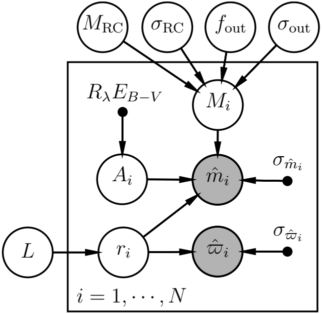

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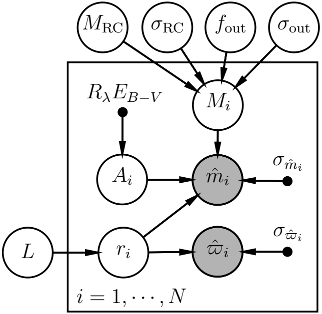
Bayesian inference



Data:
$\mathcal{D}_i = (\hat{m}_i, \sigma_{\hat{m}_i}, \hat{\varpi}_i, \sigma_{\hat{\varpi}_i}, E(B-V)_i)$
OR:
$\mathcal{D}_i = (\hat{m}_i, \hat{\varpi}_i)$

$$p(\theta_{\rm RC}, L \mid \{\mathcal{D}_i\}) \propto p(\theta_{\rm RC}, L) \prod_i p(\mathcal{D}_i \mid \theta_{\rm RC}, L, \sigma_{\hat{m}_i}, \sigma_{\hat{\varpi}_i}, E(B-V)_i)$$

$$p(\mathcal{D}_i | \ ...) = p(\hat{\varpi}_i \mid 1/r_i, \sigma_{\hat{\varpi}_i}) \times p(\hat{m}_i | \ ...)$$

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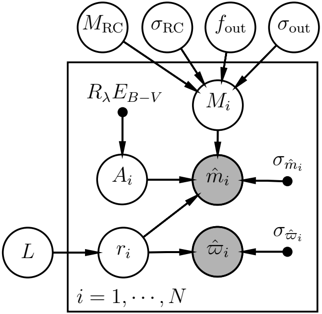

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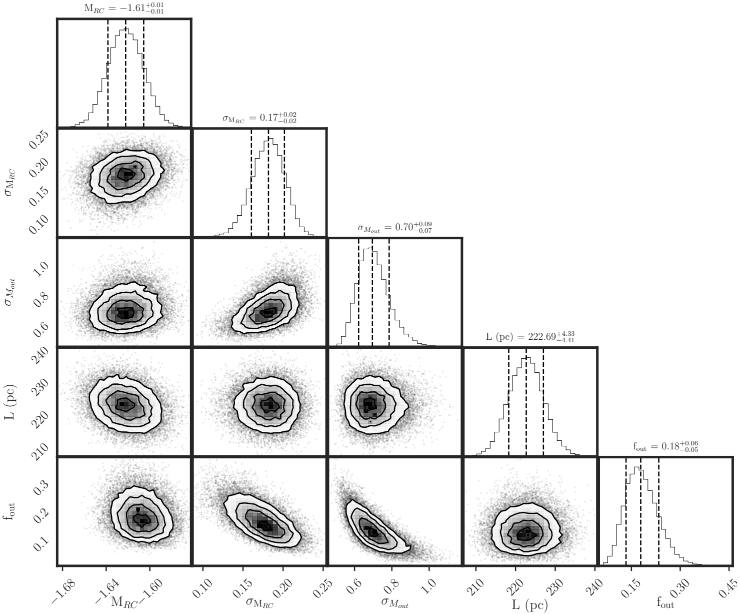
Example posterior



Radek Poleski Hawkins et al. (2017)

Results

Table 1. Red clump model parameters for the *J*, *H*, *K*, *G*, *W1*, *W2*, *W3* and *W4* bands.

Band	M_{RC} (mag)	$\sigma_{M_{RC}}$ (mag)	$\sigma_{M_{out}}$ (mag)	L (pc)	f_{out}	N	$R_\lambda = \frac{A_\lambda}{E(B-V)}$
G	+0.44 ± 0.01	0.20 ± 0.02	0.75 ± 0.08	215.6 ± 4.2	0.18 ± 0.04	972	2.85
J	–0.93 ± 0.01	0.20 ± 0.02	0.72 ± 0.09	213.5 ± 4.0	0.13 ± 0.05	972	0.72
H	–1.46 ± 0.01	0.17 ± 0.02	0.71 ± 0.09	$213.3^{+4.1}_{-3.9}$	0.18 ± 0.05	972	0.46
K_s	–1.61 ± 0.01	0.17 ± 0.02	$0.70^{+0.10}_{-0.08}$	222.7 ± 4.3	0.18 ± 0.05	972	0.30
W1	–1.68 ± 0.02	0.10 ± 0.04	$0.73^{+0.12}_{-0.09}$	231.5 ± 4.8	0.15 ± 0.04	936	0.18
W2	–1.69 ± 0.02	0.20 ± 0.03	0.84 ± 0.10	237.8 ± 4.8	0.15 ± 0.04	934	0.16
W3	–1.67 ± 0.01	0.17 ± 0.02	0.74 ± 0.08	228.3 ± 4.6	0.18 ± 0.05	936	0.16
W4	–1.76 ± 0.01	0.16 ± 0.02	$0.73^{+0.09}_{-0.07}$	221.1 ± 4.5	0.18 ± 0.05	910	0.11

Note. The bandpass is shown in column 1 while the absolute magnitude and dispersion in the absolute magnitude of the RC and 'contaminate' population in that bandpass is listed in columns 2, 3, 4 and 5, respectively. The inferred scalelength of the distance prior is tabulated in column 6 and the contaminate fraction, f_{out} can be found in column 7. The number of stars used in the inference and the assumed extinction coefficient for each band is tabulated in column 8 and 9, respectively.

Output

Sampling of:

$\theta_{\mathrm{RC}} = \{M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}}, \sigma_{M_{out}}, f_{\mathrm{out}}\}$

L

$\{r_i\}$

Output

Sampling of:
$\theta_{\mathrm{RC}} = \{M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}}, \sigma_{M_{out}}, f_{\mathrm{out}}\}$
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Quote 1:
"We only select those stars which have an inferred probability of belonging to the RC population that is larger than or equal to 80 per cent."

Quote 2:
"Probable RC stars are defined as those which have probabilities of being attributed to the RC component greater than or equal to 80 per cent. In this case, the probability for each star belonging to the RC is computed for every MCMC chain and the median is taken."

Output

Sampling of:
$\theta_{\mathrm{RC}} = \{M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}}, \sigma_{M_{out}}, f_{\mathrm{out}}\}$
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What else they calculated?

Output

Sampling of:
$\theta_{\mathrm{RC}} = \{M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}}, \sigma_{M_{out}}, f_{\mathrm{out}}\}$
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What else they calculated? I'm guessing:
$M_i = \hat{m}_i - 5\log_{10}(r_i) + 5 - A_i$

Output

Sampling of:
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L
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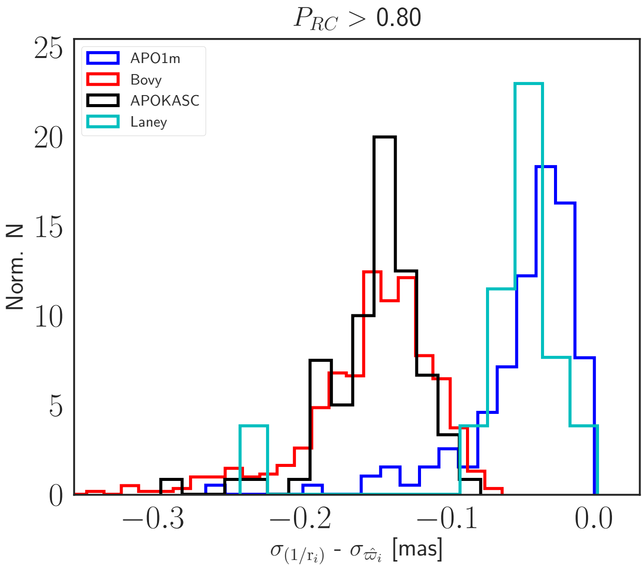
Which allows calculating probability that given star belongs to RC:
$p(\mathrm{RC}, i) = \frac{(1-f_{\mathrm{out}})\mathcal{N}(M_i \; | M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}})}{(1-f_{\mathrm{out}})\mathcal{N}(M_i \; | M_{\mathrm{RC}}, \sigma_{M_{\mathrm{RC}}}) + f_{\mathrm{out}}\mathcal{N}(M_i \; | M_{\mathrm{RC}}, \sigma_{M_{\mathrm{out}}})}$

Error shrinkage



Critique

- Inconsistent notation throughout the paper.
- Unclear $p(RC, i)$.
- Hard to read histograms.
- No ticks on the axes.
- L and r_i values change between bands.
- Fixed R_λ.

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