Anderson-Darling test for Spitzer systematic errors in microlensing parallax

Koshimoto N. and Bennett, D.P., 2020, The Astronomical Journal, 160:177.

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Introduction

Spitzer microlensing events and parallax so far ...

- Clear magnification
- Clear baseline

But, also reported ...

- Large light curve deviation
- Large dispersion or retrograde orbit of the lens

Let’s figure out the cause of errors!

Microlensing parallax from *Spitzer*

Galactic models

- Sumi et al. (2011)
- Bennett et al. (2014)
- Zhu et al. (2017)

Anderson-Darling test

Established by ...

- Anderson, T.W. and Darling, D.A. (1952, 1954)
- Stephens, M.A. (1974, and so forth)

Basic concept :

- Is a given sample from a given probability distribution?

Similar statistical test :

- Cramér–von Mises test
- Kolmogorov–Smirnov test
- Shapiro-Wilk Normality test

Anderson-Darling test

Basic formula :

$$Discrepancy = n \int_{-\infty}^{+\infty} [F_n(x) - F(x)]^2 \, \psi[F(x)] \, dF$$

n	: Number of elements in a sample
$F_n(x)$	: Cumulative distribution of the element (n)
$F(x)$	: Cumulative distribution of the model
$\psi[F(x)]$	: Weight function

$$\psi[F(x)] = \{F(x)[1 - F(x)]\}^{-1}$$

$$A^2 = n \int_{-\infty}^{+\infty} \frac{[F_n(x) - F(x)]^2}{F(x)[1 - F(x)]} \, dF$$

Anderson-Darling Statistics

Anderson-Darling test

If samples are from any hypothesized distribution ...

$$\begin{cases} F_n(x) \rightarrow F(x) \\ F_n : F \rightarrow F_{uniform} \end{cases} = \text{linear}$$

For observed data *Y* and model distribution *F* :

$$A^2 = -n - \sum_{i=1}^{n} \frac{2i-1}{n} \left[\ln\left(F\left(Y_i\right)\right) + \ln\left(1 - F\left(Y_{n+1-i}\right)\right) \right]$$

where $Y = \{Y_1 < Y_2 < \ldots < Y_n\}$

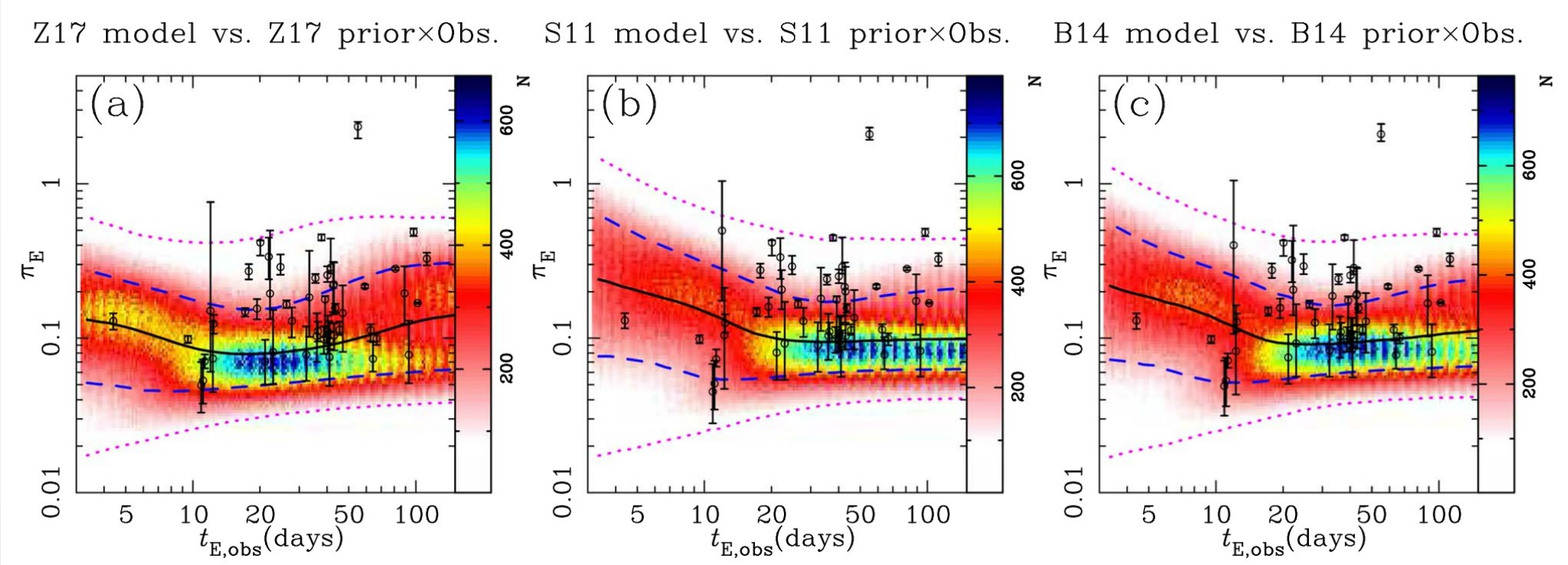
Spitzer parallax error & AD test

Koshimoto & Bennett (2020) have ...

[Samples]
50 microlensing events
observed by
OGLE & *Spitzer*

AD test

[Models]
S11 (Sumi et al. 2011)
B14 (Benett et al. 2014)
Z17 (Zhu et al. 2017)



Spitzer parallax error & AD test

"Position" of a sample when arranged w.r.t. probability :

$$P_{Gal}\left(\pi_{E,prior} \gtrsim \pi_E | t_{E,obs}\right) \equiv \int_{\pi_E}^{\infty} \Gamma_{Gal}\left(\pi_E^{'} | t_{E,obs}\right) d\pi_E^{'}$$

$\pi_{E,prior}$: π_E *from the model*
$\pi_E \mid t_{E,obs}$: Sample π_E w.r.t. t_E
Γ_{gal} : Event rate of the model w.r.t. π_E & t_E

Cumulative density function :

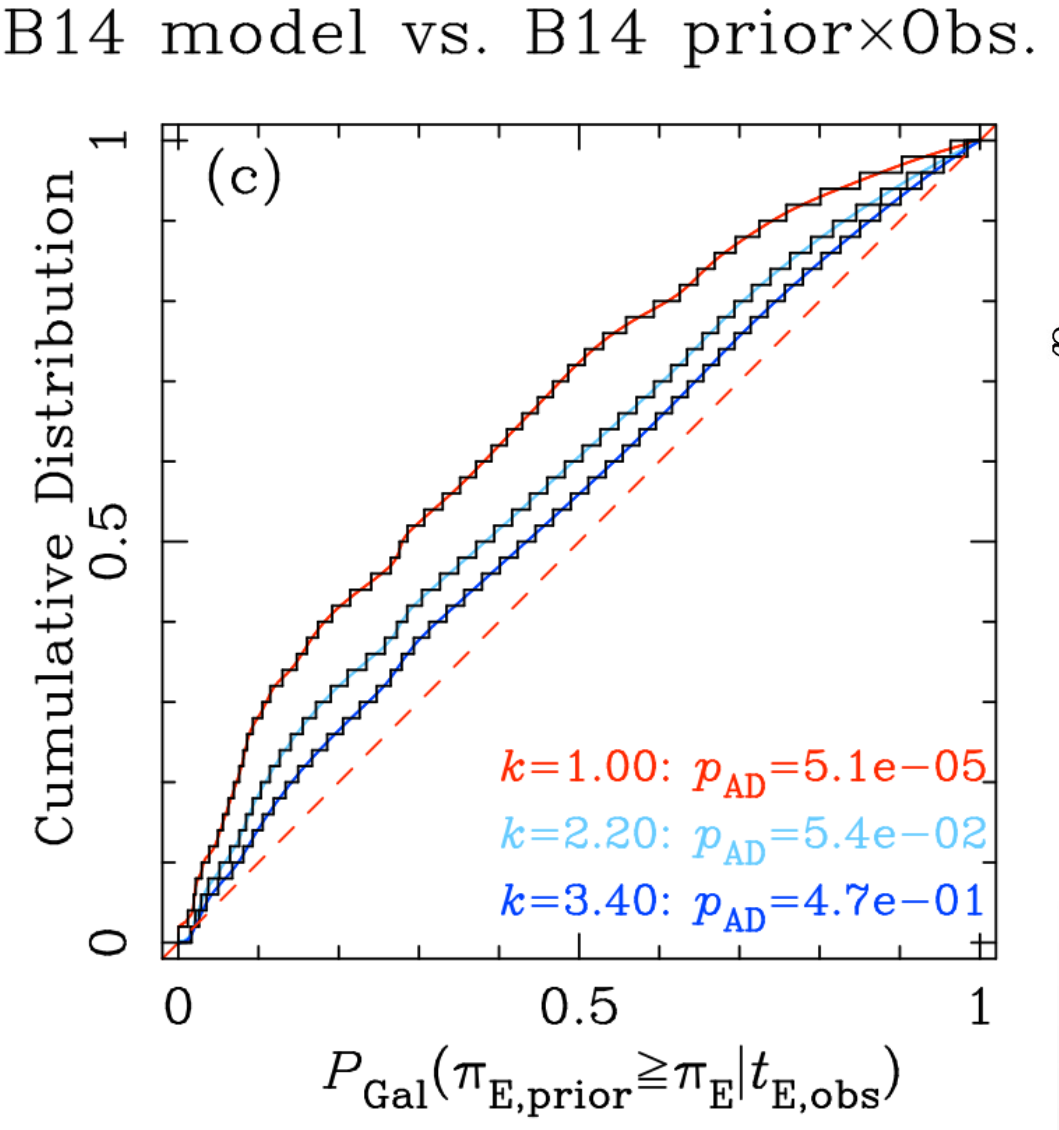
$$G_{post}\left(P_{Gal}\right) \equiv \int_{0}^{P_{Gal}} \sum_{i=1}^{N_{eve}} \frac{g_{post,i}\left(P_{Gal}^{'}\right)}{N_{eve}} dP_{Gal}^{'}$$

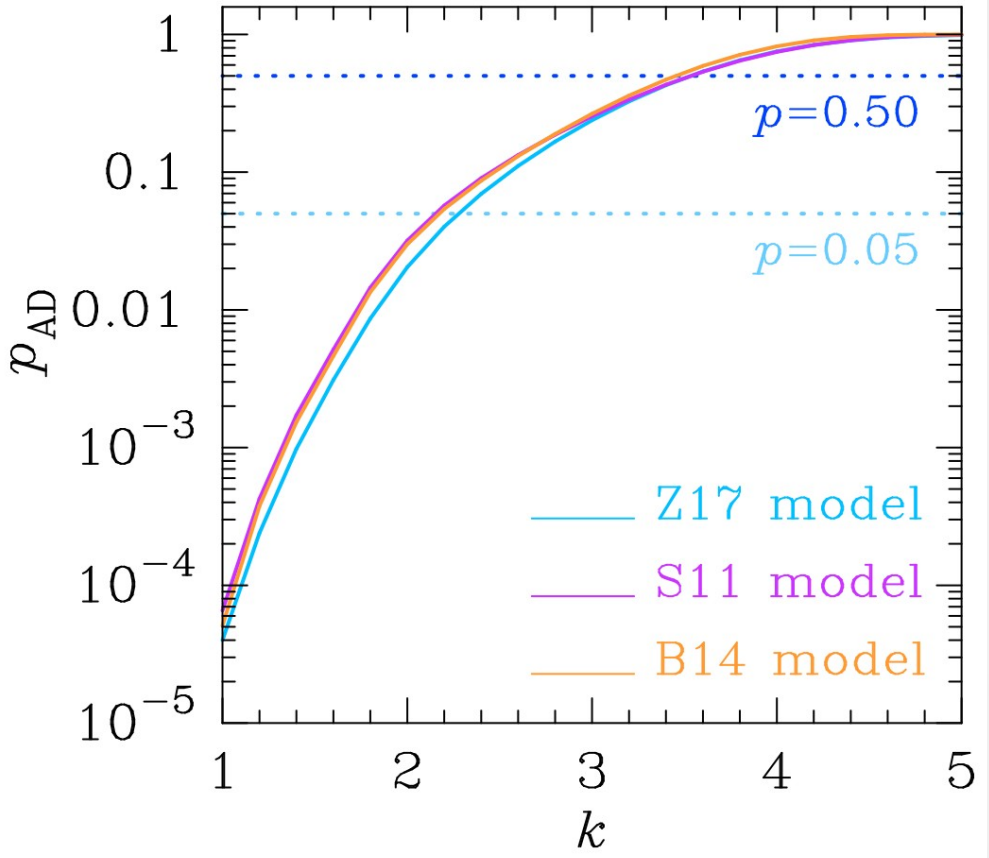
→ Linear?

N_{eve} : Number of samples (= 50 events)
g_{post} : Distribution of inverse percentile

Spitzer parallax error & AD test

AD test results :

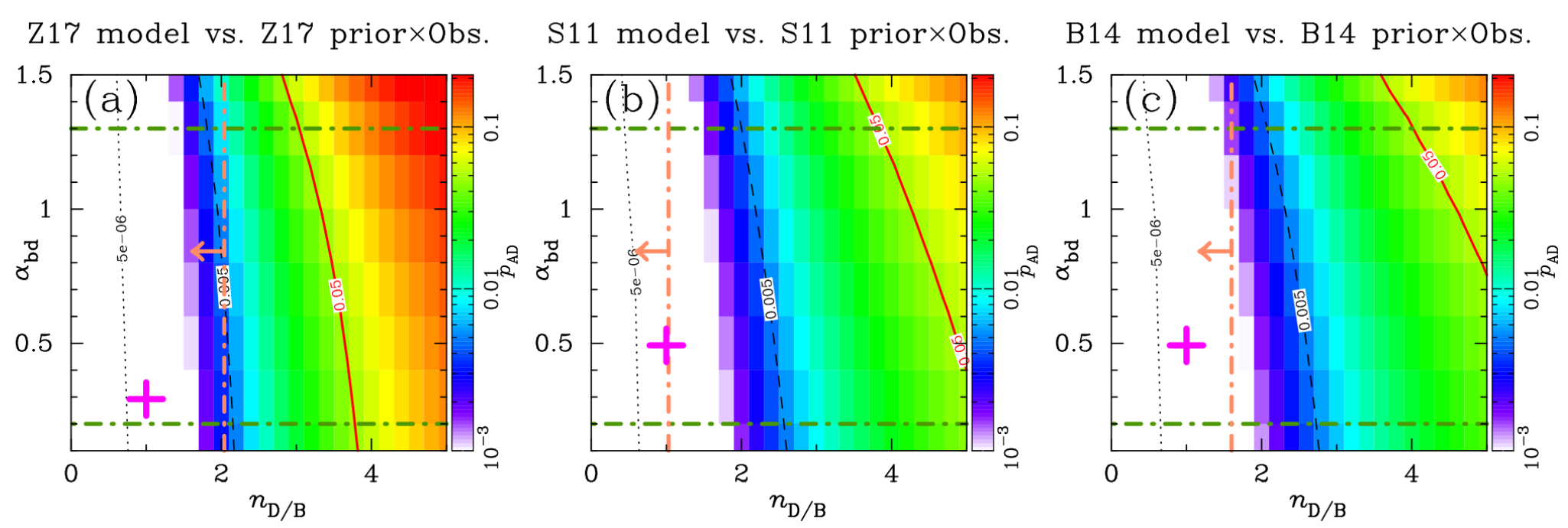




Model (k=1)	A^2	p_{AD}
Z17	9.01	4.0e-5
S11	8.54	6.6e-5
B14	8.78	5.1e-5

Spitzer parallax error & AD test

Where does the discrepancy come from?



Model	$n_{D/B.th}$
Z17	3.1
S11	3.9
B14	4.0

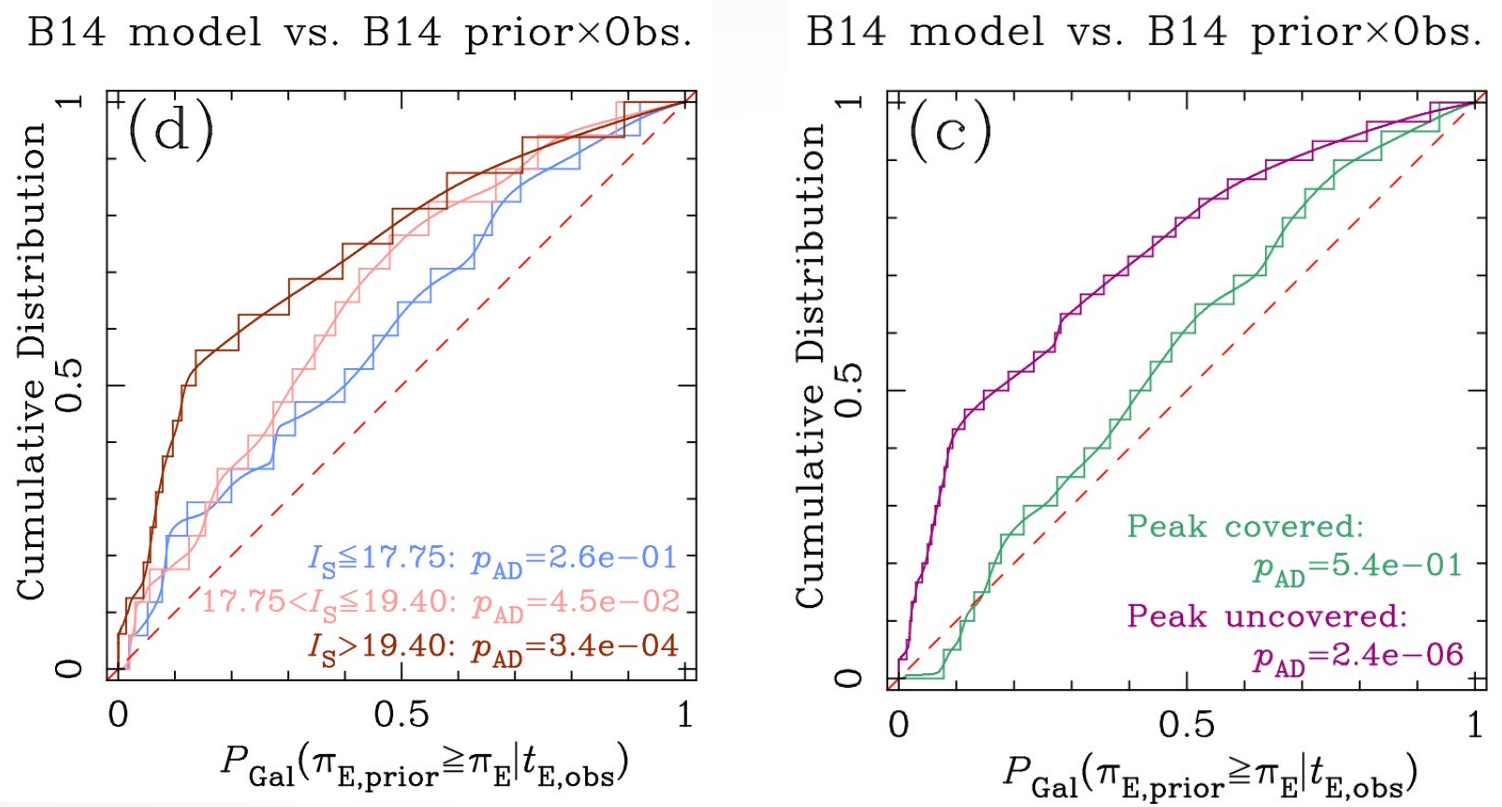
$n_{D/B}$: Change of the disc-bulge mass ratio w.r.t. the default ratio

α_{bd} : Slope of the brown dwarf initial mass function

* Theoretically required $n_{D/B}$ for p_{AD}>0.05, 0.2<α_{bd}<1.3 at k=1

Spitzer parallax error & AD test

Where does the discrepancy come from?



Model (k=1)	$p_{AD}(I_S)$ <17.75, 17.75-19.40, >19.40	p_{AD}(peak) covered, not covered	p_{AD}(phot_error) obvious, not obvious
Z17	0.13, 0.026, 1.4e-3	0.098, 2.6e-5	1.5e-4, 0.029
S11	0.27, 0.054, 3.7e-4	0.63, 2.7e-6	1.2e-4, 0.046
B14	0.26, 0.045, 3.4e-4	0.54, 2.4e-6	1.3-e4, 0.035

Summary

Anderson-Darling test

- χ^2-like analysis for cumulative distribution (F_n v.s. F)
- If samples are from model distribution, F_n : F → linear

Spitzer systematic error and AD test

- Poor sample-model distribution agreement: $p_{AD} \leq$ 6.6e-5
- Possible to reduce discrepancy: bright source, peak coverage, photometric error estimation
- Still F_n : F > linear case → Unknown systematic error

Reference

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- Dodge, Y., 2008, “The Concise Encyclopedia of Statistics,” *Springer New York*, pp.12-14.
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