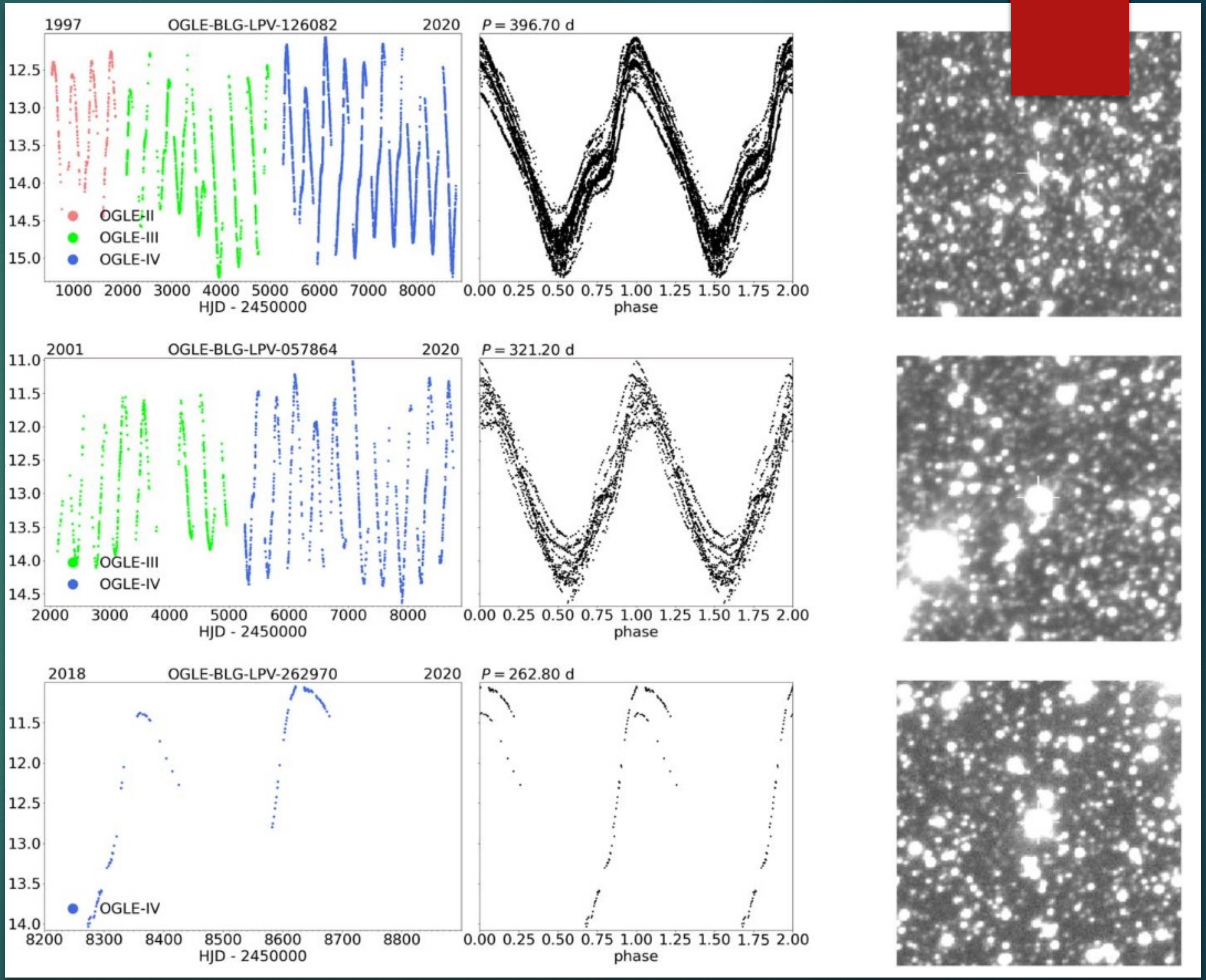
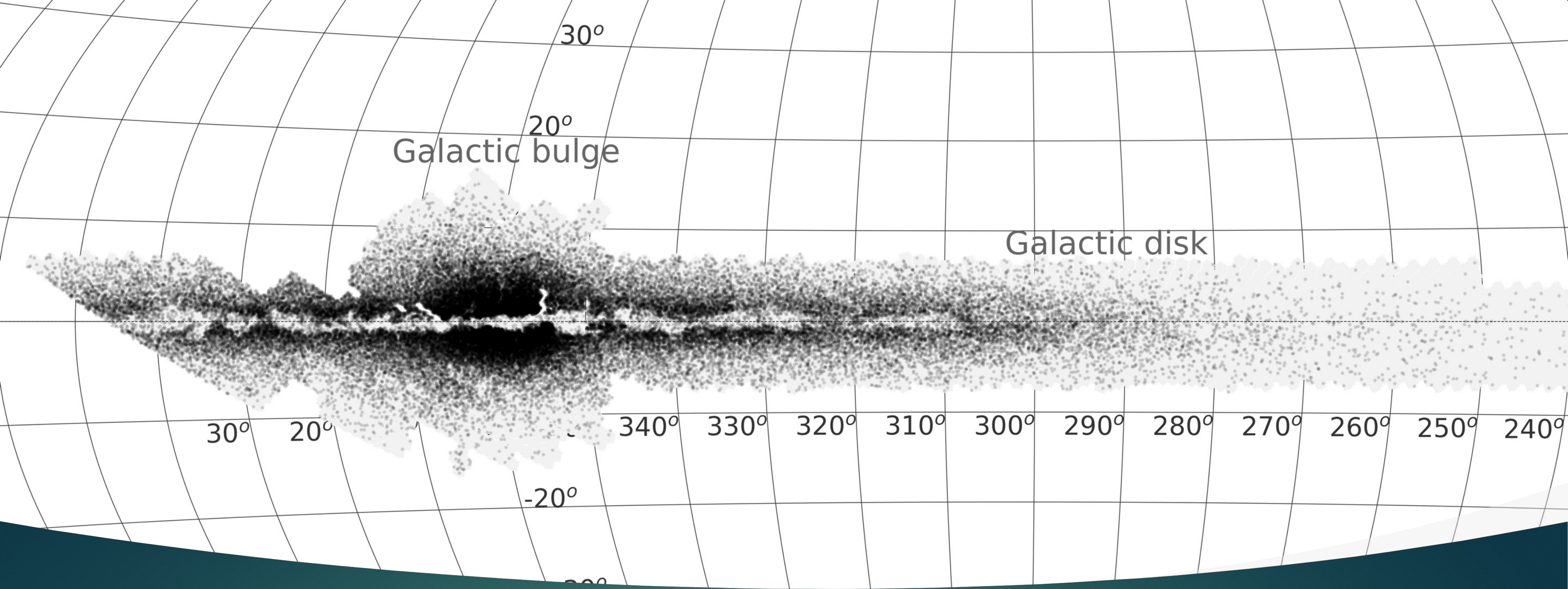
A Three dimensional Map of the Milky Way Using 66,000 Mira Variable Stars

IWANEK, P., ET AL., APJS, 264, 20 (2023)

A few words about Miras

- stars at the end of their lifes, just before planetary nebula and white dwarf stages
- high-amplitude, fundamental mode pulsators
- pulsation periods from 80 to over 1000 days
- amplitudes larger than 2.5 mag in visual bands
- amplitudes decreases with the wavelength



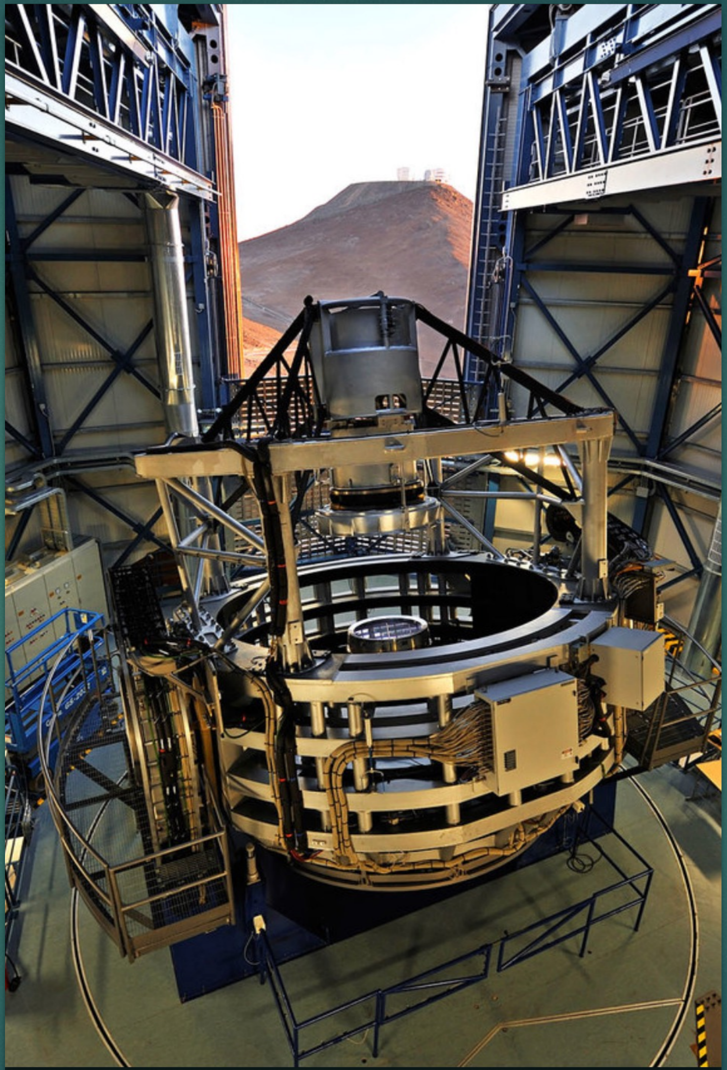


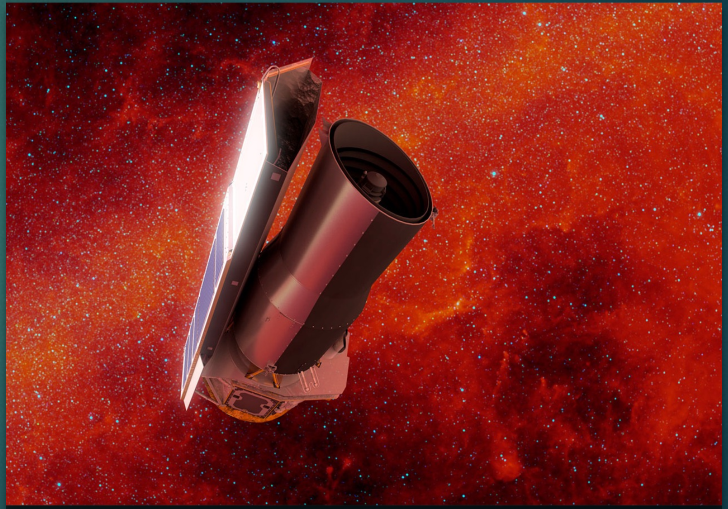
We based the analysis on the catalog of 66,000 Mira stars discovered in the OGLE data (periods, amplitudes, coordinates, mean magnitudes)

IWANEK, P., ET AL., APJS, 260, 46 (2022)

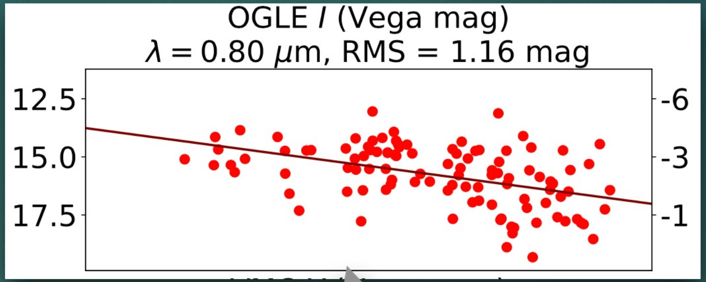
Very important – mid-infrared data

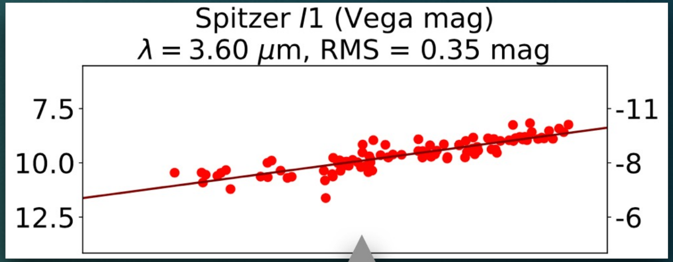
- We cross-matched the catalog with WISE/Spitzer/VMC databases
- We used in total observations in 12 bands



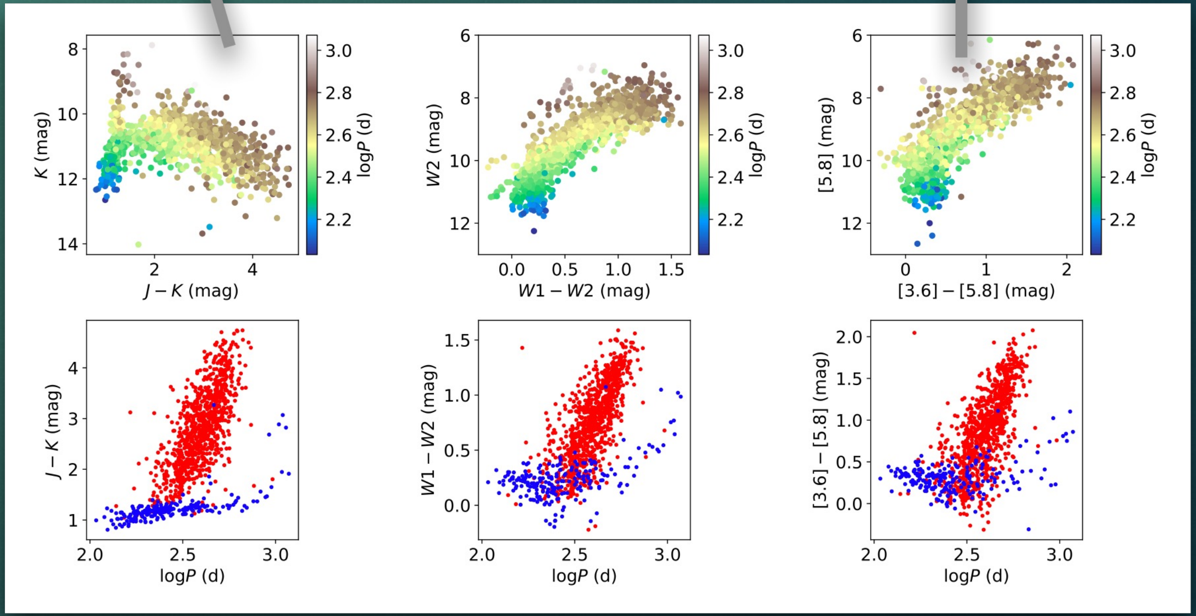


Very important – mid-infrared data



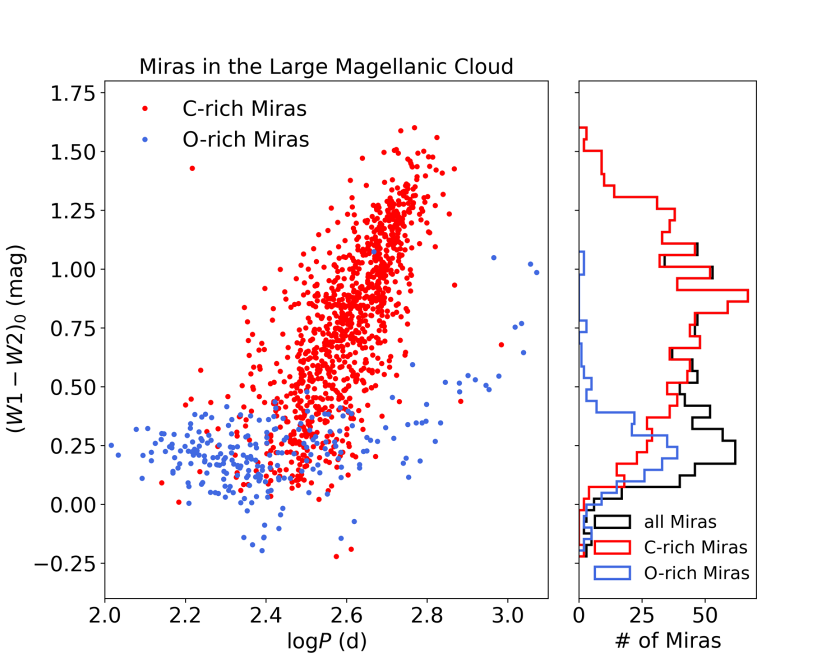


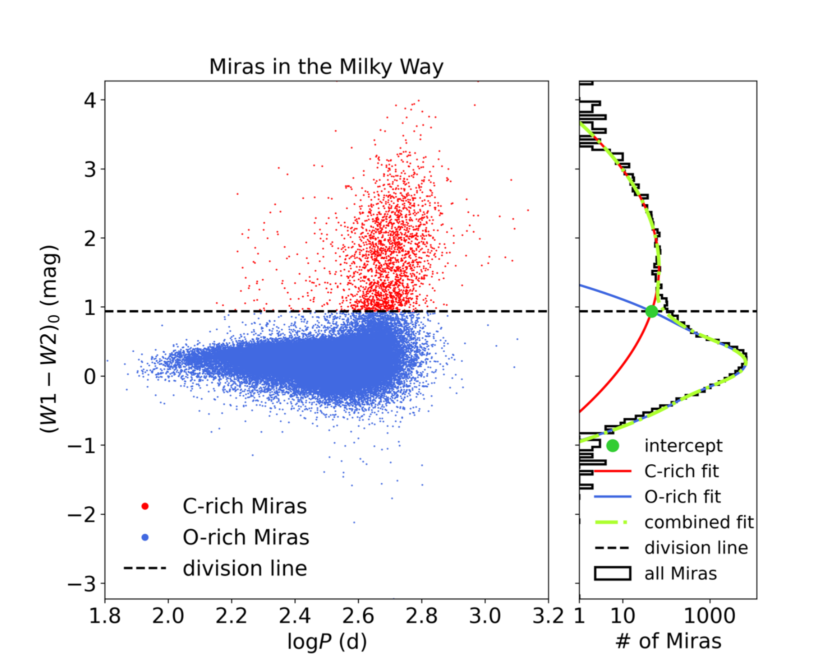
- smaller amplitudes in the mid-infrared wavelengths
- smaller extinction
- more accurate distance measurements



Division into Oxygen- and Carbon-rich

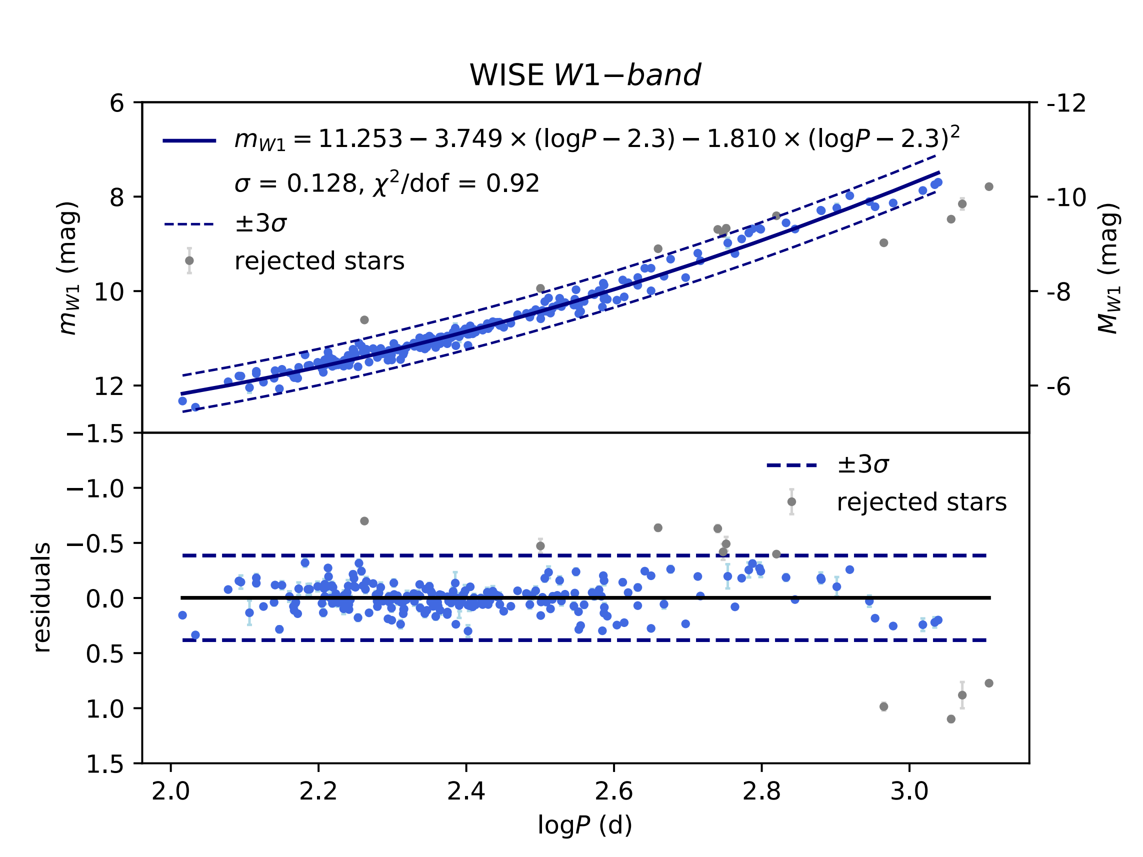
- similar distribution/shape as the Large Magellanic Cloud (LMC) Miras
- we divided Miras into Oxygen- and Carbon-rich using Gaussian Mixture Models with three components
- first cut –
  only O-rich Miras





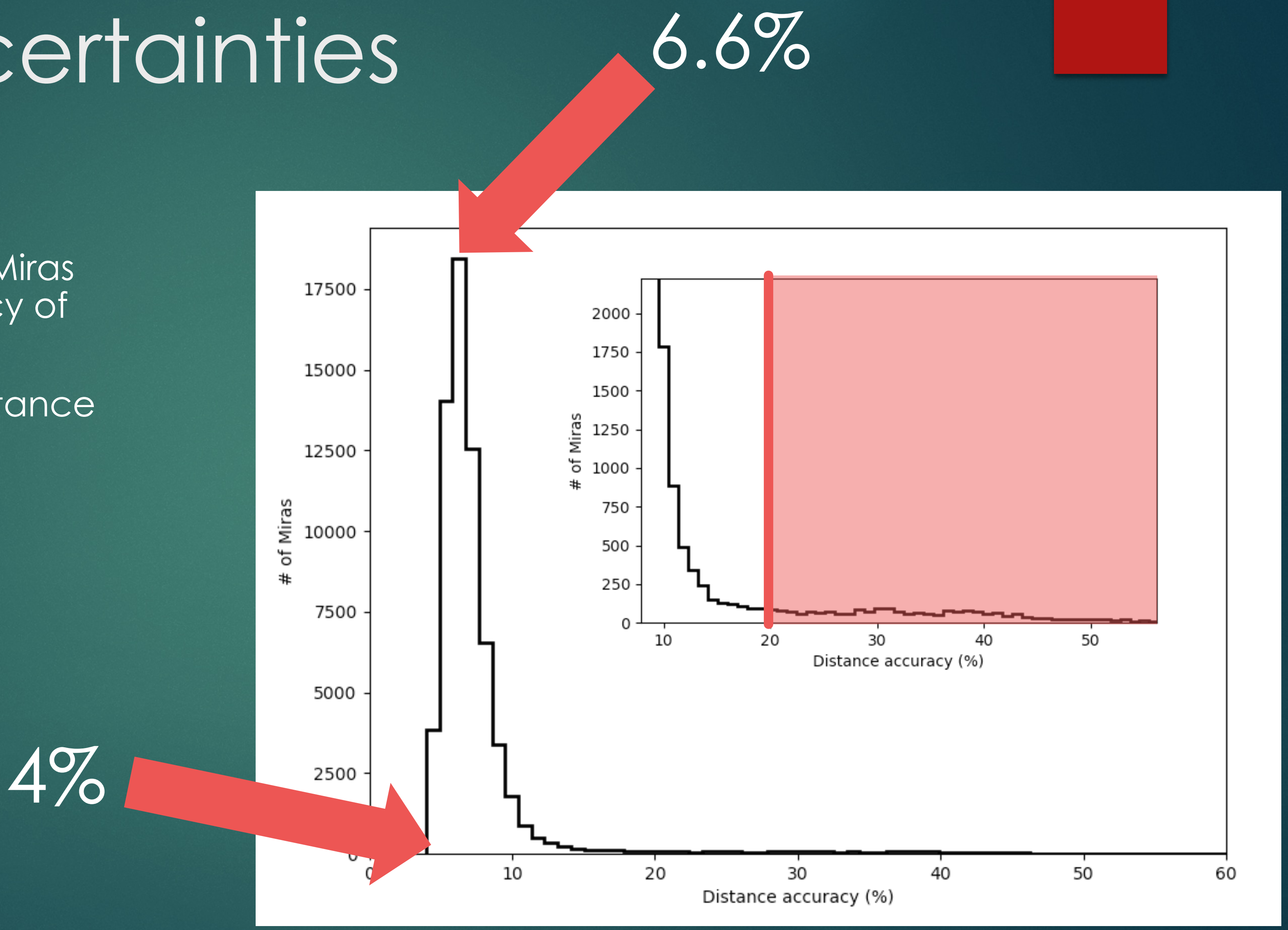
Distance measurements

- we used period-luminosity relations (PLRs) from Iwanek, P., et al., ApJ, 919, 99 (2021)
- PLRs calibrated on the distance to the LMC (49.59 ± 0.09 ± 0.54 kpc; Pietrzyński et al., 2019)



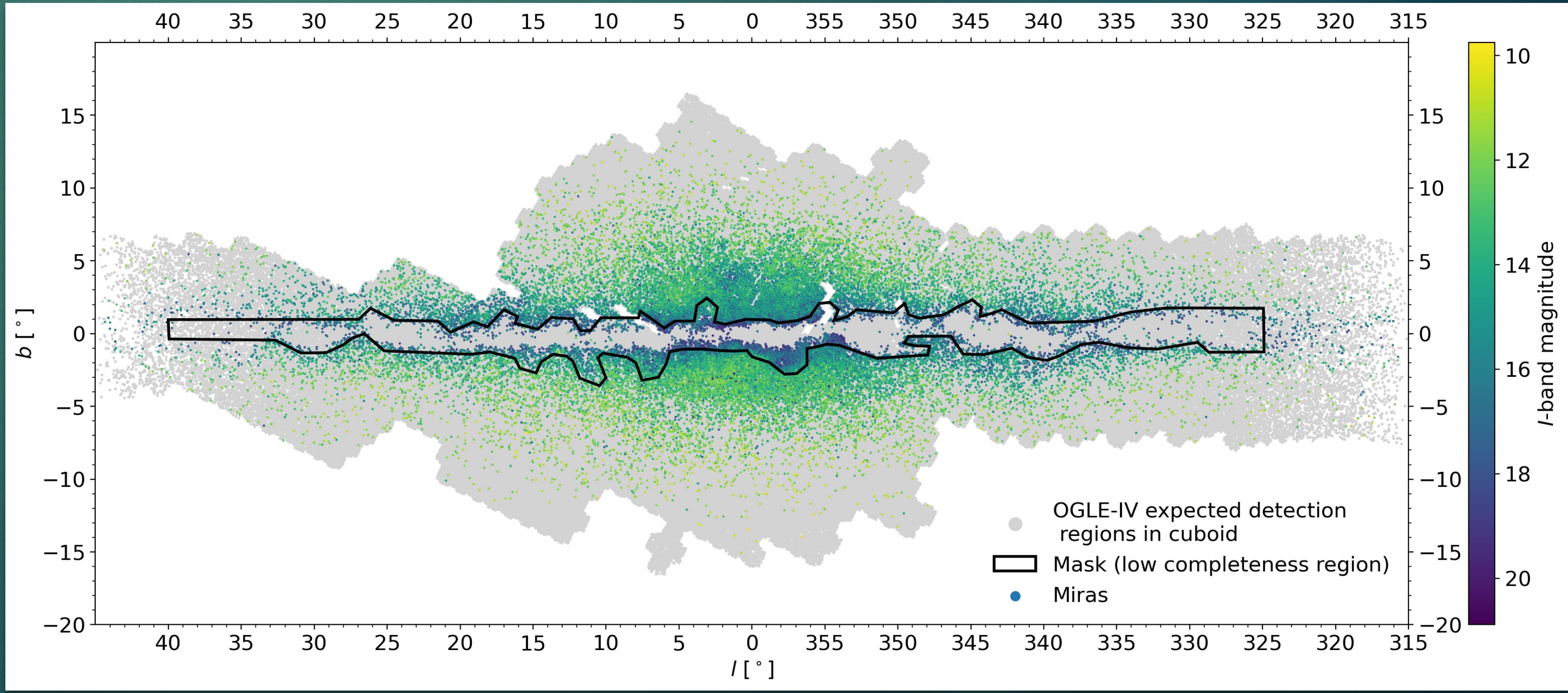
Distance uncertainties

- distances to O-rich/C-rich Miras measured with an accuracy of ~5%/12%
- second cut – Miras with distance accuracy better than 20%



The most interesting part of the Milky Way – Galactic bulge

- transformation to the Cartesian coordinates
- cut of the three-dimensional cuboid with the size of 8 x 8 x 4 kcp
- cutting out the area with low completeness



Final sample

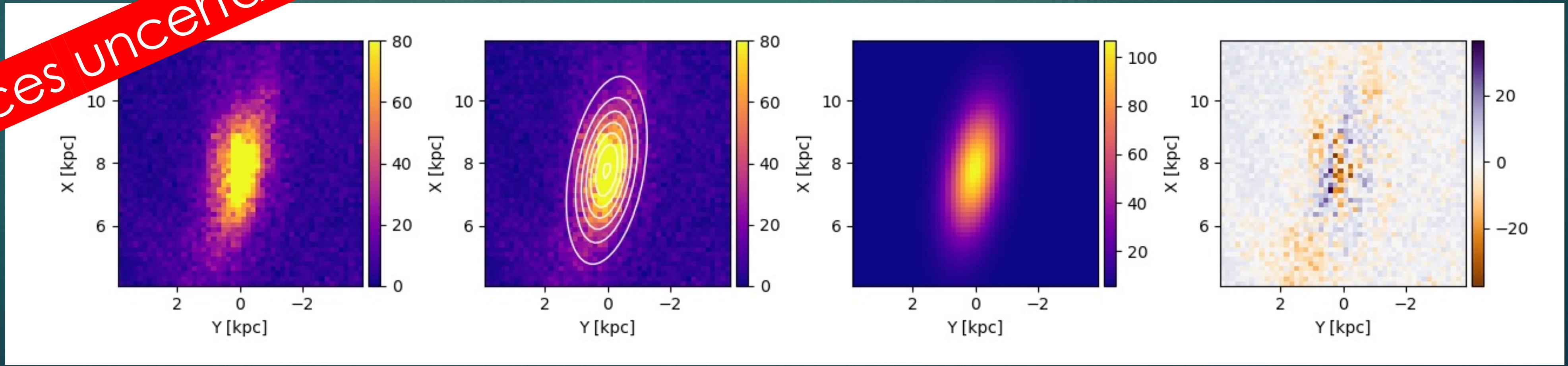
31,992 Miras

First attempt to model the Galactic bulge

- model: three-dimensional Gauss
- weakness: fitting without distance uncertainties

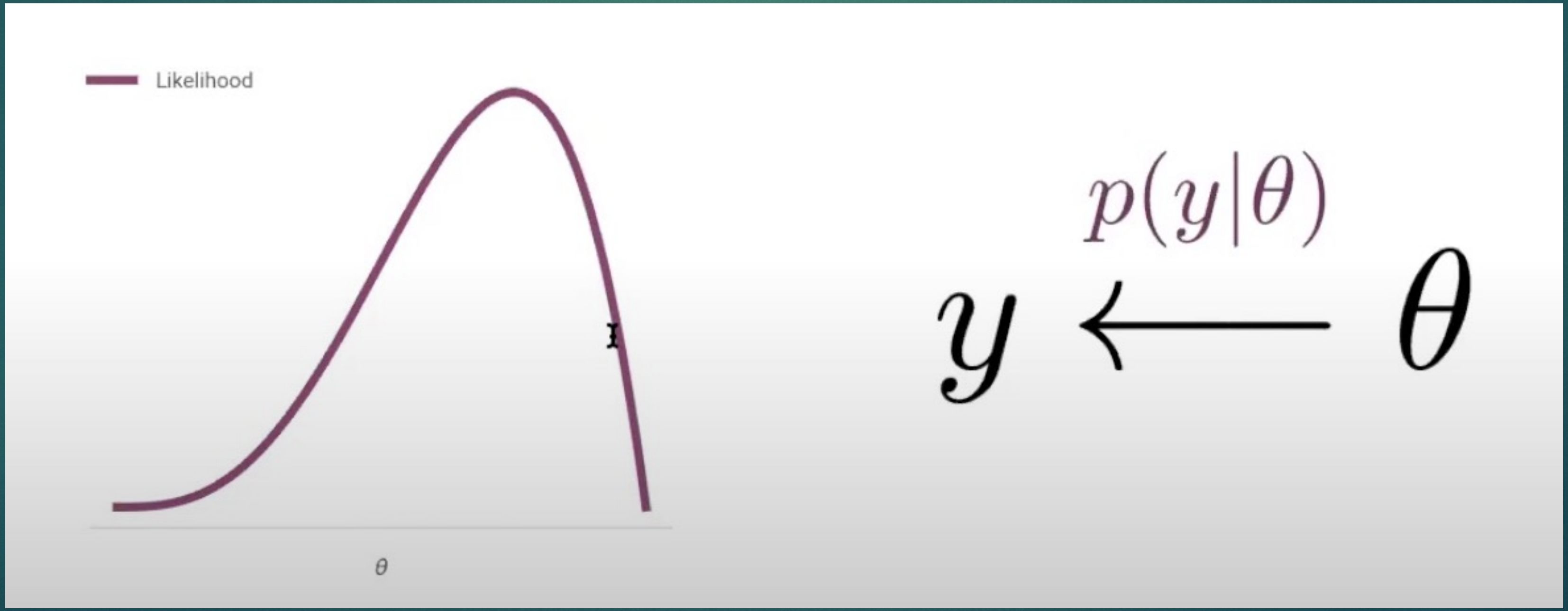
```python
def model(amplitude, xo, yo, sigma_x, sigma                 set, hz,  X, Y, Z):

    xo = float(xo)
    yo = float(yo)

    dx = X-xo
    dy = Y-yo

          eta)**2)/(2*sigma_x**2) + (np.sin(theta)**2)/(2*sigma_y**2)
          in(2*theta))/(4*sigma_x**2) + (np.sin(2*theta))/(4*sigma_y**2)
          np.sin(theta)**2)/(2*sigma_x**2) + (np.cos(theta)**2)/(2*sigma_y**2)

    aa = -a
    bb = -2. * b
    cc = -c
    dd = 1. / hz

    return offset + amplitude*np.exp(aa*dx**2+bb*dx*dy+cc*dy**2+dd*np.abs(Z))
```

Distances uncertainties are significant and should be taken into account!



Idea: Bayesian inference

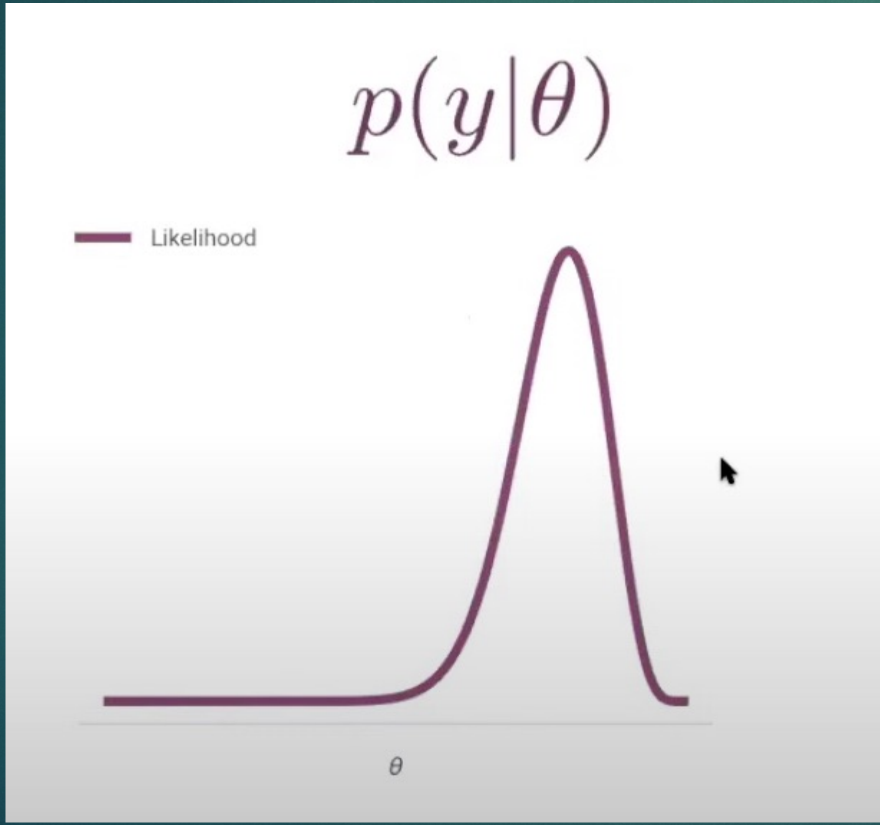
- **inference** – all about understanding our data
- likelihood function encodes our assumptions about the data generating process
- likelihood function tells us which values of the parameters are more **„consistent”** with the data that we have

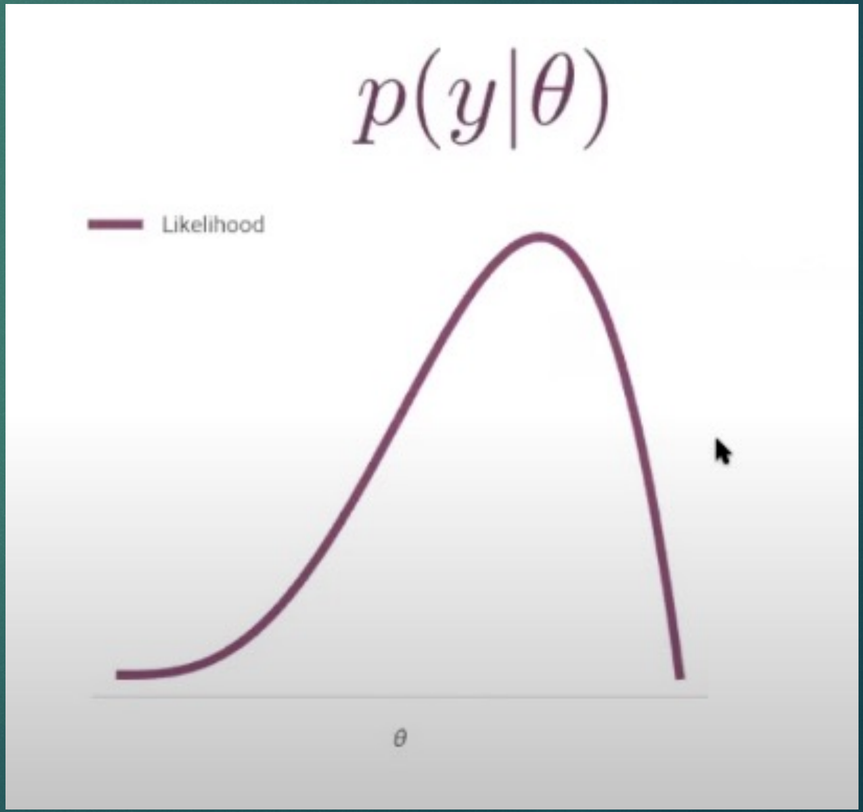


source: https://youtu.be/38yOWMMCeMk

Idea: Bayesian inference

- **informative** data results in a tight likelihood function (tight ranges of parameters)
- **uniformative** data results in a diffuse likelihood (wide ranges of parameters)





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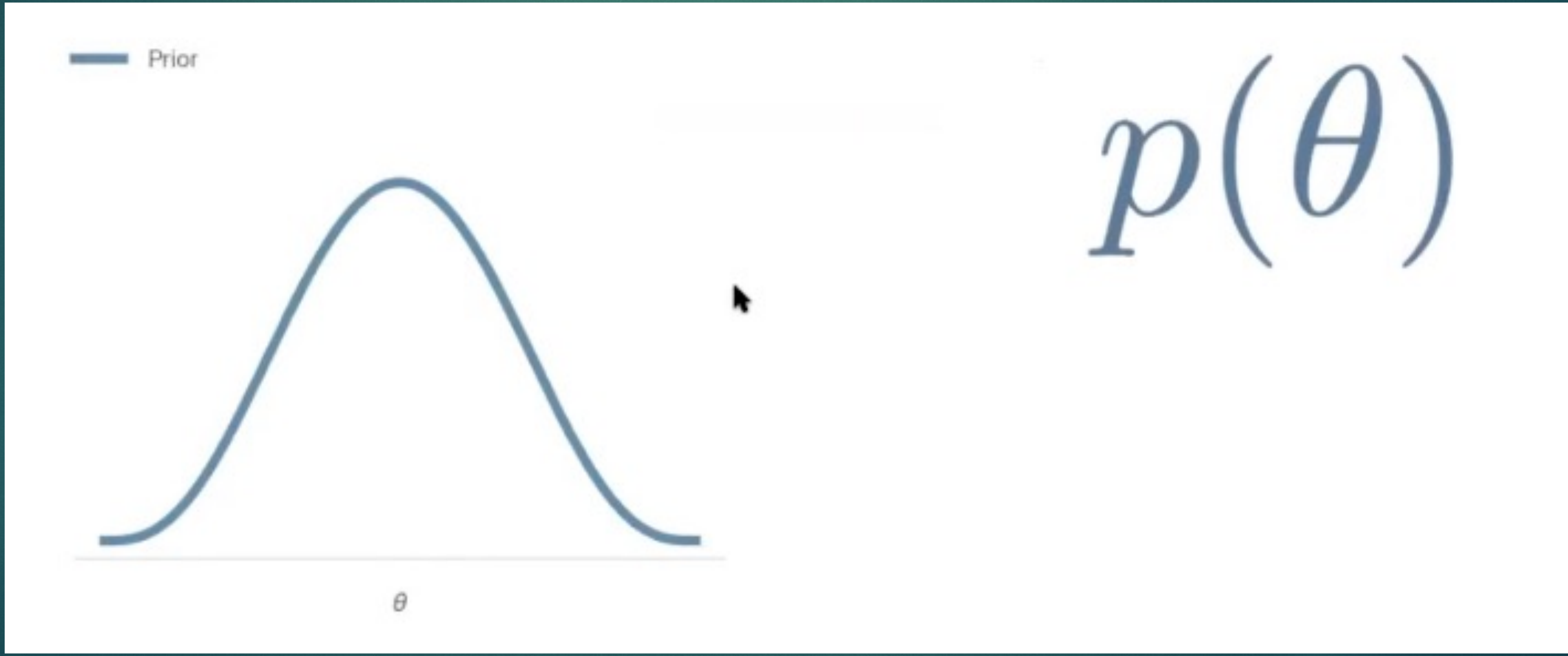
Idea: Bayesian inference

- **likelihood** is a half of the story
- Bayesian inference requires **prior** knowledge about parameters – this is encoded into probability distribution called **prior**



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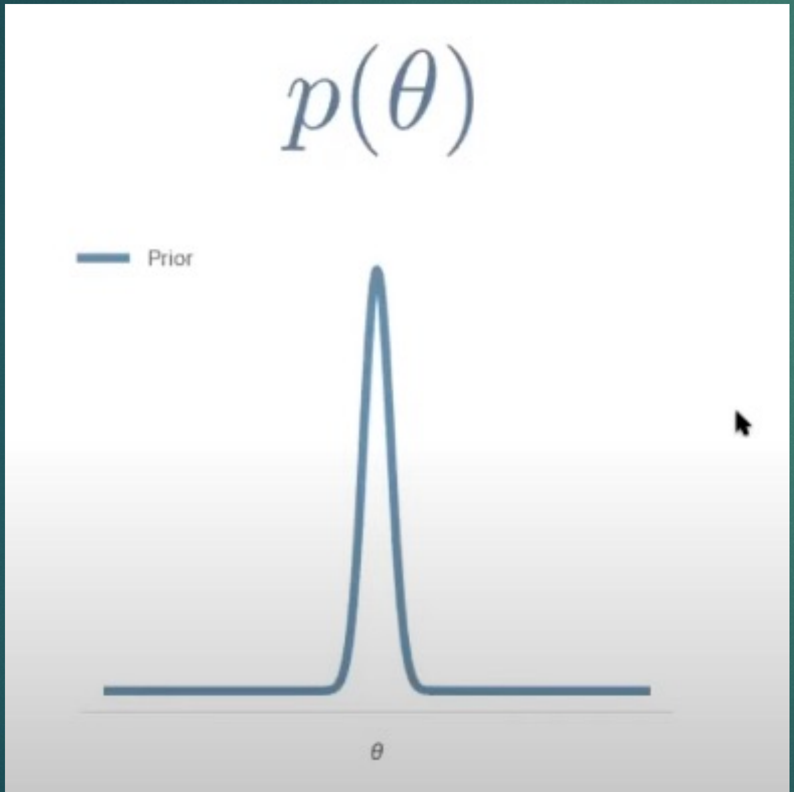
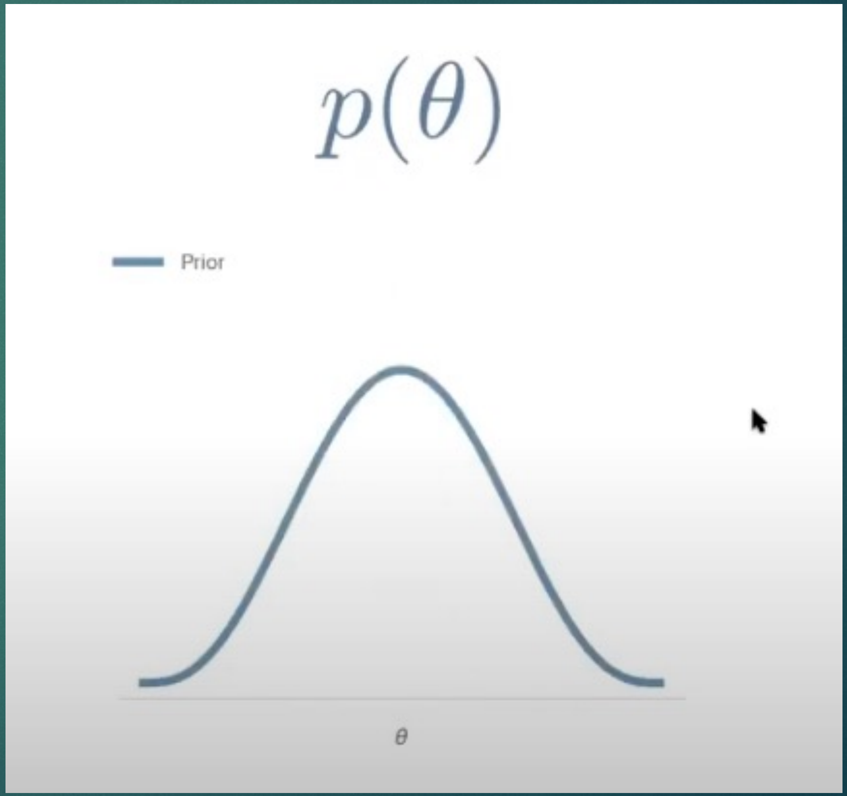
Idea: Bayesian inference

- **tigh prior** results in high confidence and small uncertainties
- **diffuse** results in poor confidence and high uncertainties





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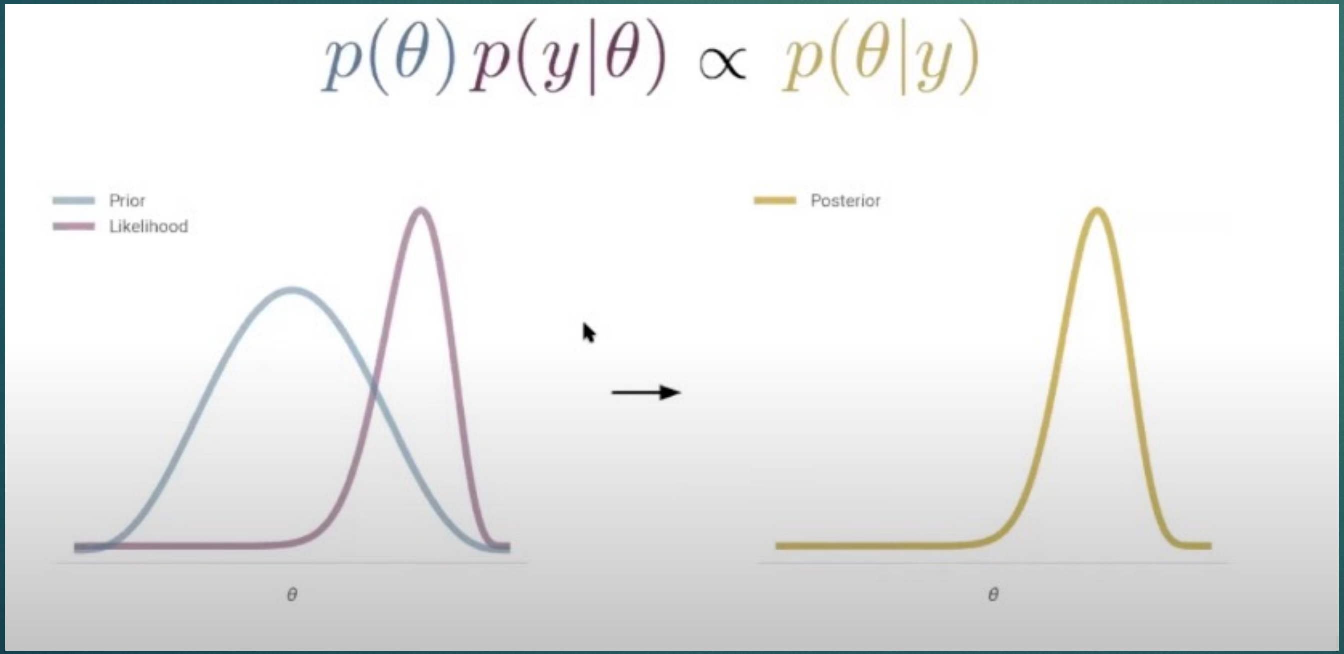
Idea: Bayesian inference

- **Bayesian inference** combine these two pieces of information
- Using **Bayes'** rule, we obtain **posterior** knowledge – the all what we know about the parameters having the prior knowledge and the data that we collected
- final goal: we want to calculate **expectation values** which is usually complicated, high-dimensional integral

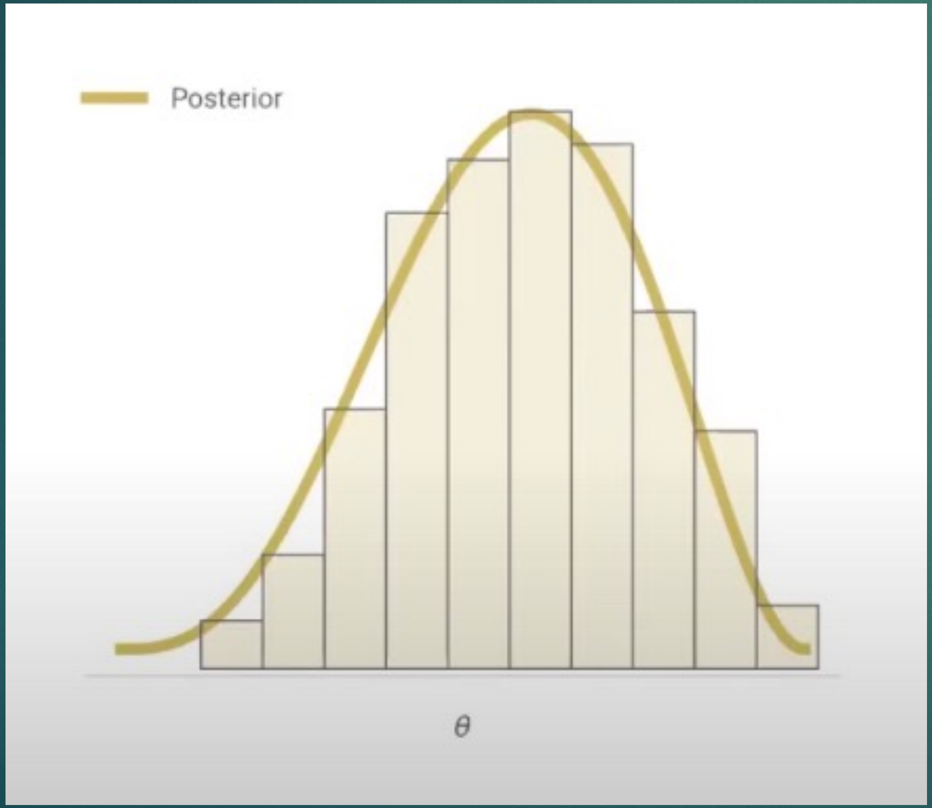


$$\mathbb{E}[f(\theta)] = \int f(\theta)p(\theta|y)d\theta$$

source: https://youtu.be/38yOWMMCeMk

Idea: Bayesian inference

- any expectations can be approximated with samples and **MCMC**



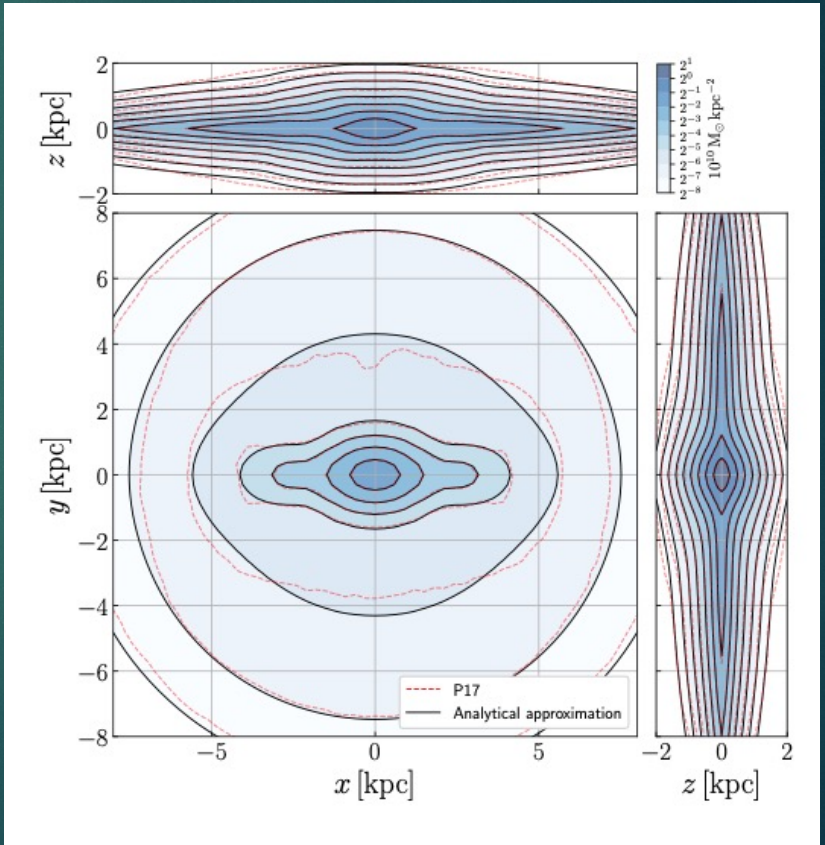
$$\mathbb{E}\left[f(\theta)\right] = \int f(\theta)p(\theta|y)d\theta$$

source: https://youtu.be/38yOWMMCeMk

Model

- 39-parameter model of the Galactic bar containing X-shape structure (Sormani et al., 2022)
- the origin of the model in the (X, Y, Z) = (0, 0, 0) kpc
- no information about total stellar mass
- five more parameters

$$\rho(x, y, z) = \underbrace{\rho_{\mathrm{bar},1} + \rho_{\mathrm{bar},2}}_{\text{bar}} + \underbrace{\rho_{\mathrm{bar},3}}_{\text{long bar}} + \rho_{\mathrm{disc}} \cdot$$



Likelihood

- **p** – vector of 44 parameters
- **N_{obs}** – number of Miras in the analysis (31,992)
- **N_exp** – normalization term, expected number of Miras in the OGLE fields

$$\mathcal{L}(\boldsymbol{p}) = e^{-N_{\exp}} \prod_{i=1}^{N_{\mathrm{obs}}} \rho'(X_i', Y_i', Z_i'; \boldsymbol{p}),$$

Ideas: **Hierarchical** Bayesian inference

- hierarchical models (with additional levels)
- in our case: distance uncertainties
- the likelihood can be modified into hierarchical form by including the posterior distribution of distance to each Mira and marginalizing over this distribution
- we achieved that by drawing from the normal distribution 10 distances for each star, with the distance as a mean and the distance uncertainty as a variance
- this approach multiples the sample by a factor of 10!

$$\mathcal{L}(\boldsymbol{p}) = e^{-N_{\mathrm{exp}}} \prod_{i=1}^{N_{\mathrm{obs}}} \left(\frac{1}{K} \sum_{k=1}^{K} \rho'(X'_{i,k}, Y'_{i,k}, Z'_{i,k}; \boldsymbol{p}) \right).$$

Priors

- **flat priors** with physically reasonable ranges for 43 parameter
- **Informative prior** for the inclination of the bar to the line of sight

$$p(\theta) = \mathcal{N}(\theta; 20, 3)$$

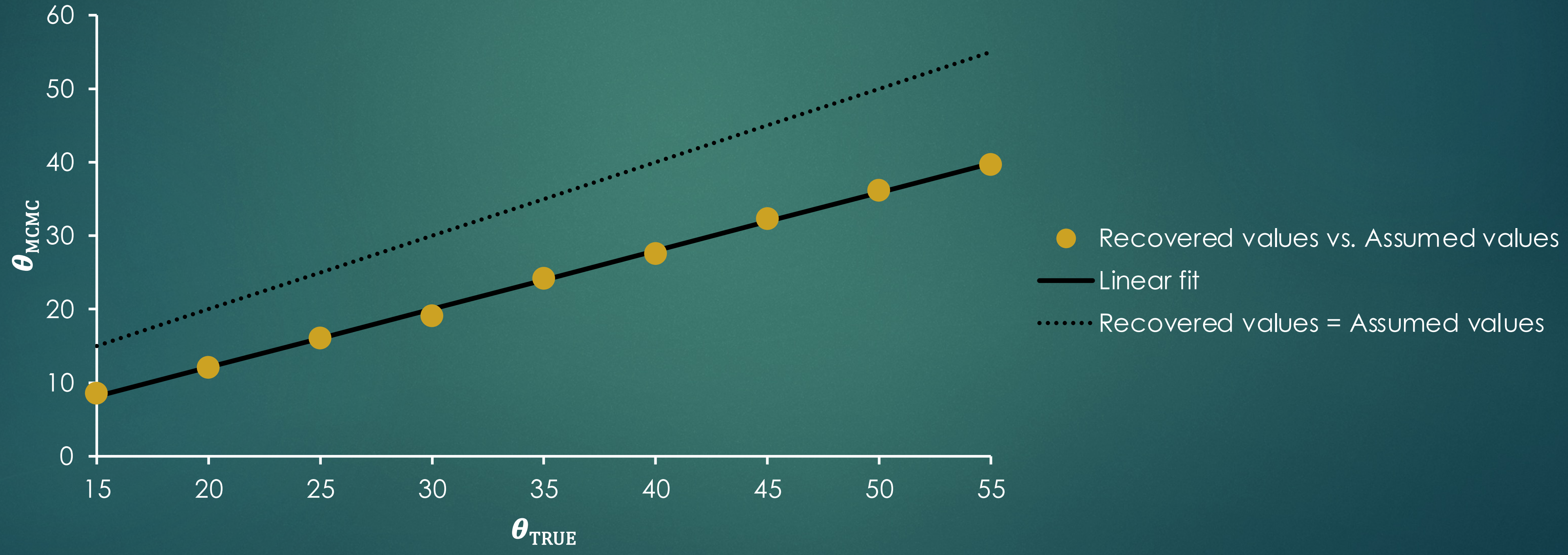
- Information about θ taken from Pietrukowicz et al., (2015)

Consistency check/reverse engineering

- we explored potential biases by assuming the model parameters and simulating mock distribution of Miras
- we fitted the model and compared the assumed parameters with the recovered values
- we added random noise to the mock distances, from the range between 4—20% of the distance
- **Result: θ is biased (systematically underestimated)**

$\boldsymbol{\theta}$ bias exploring

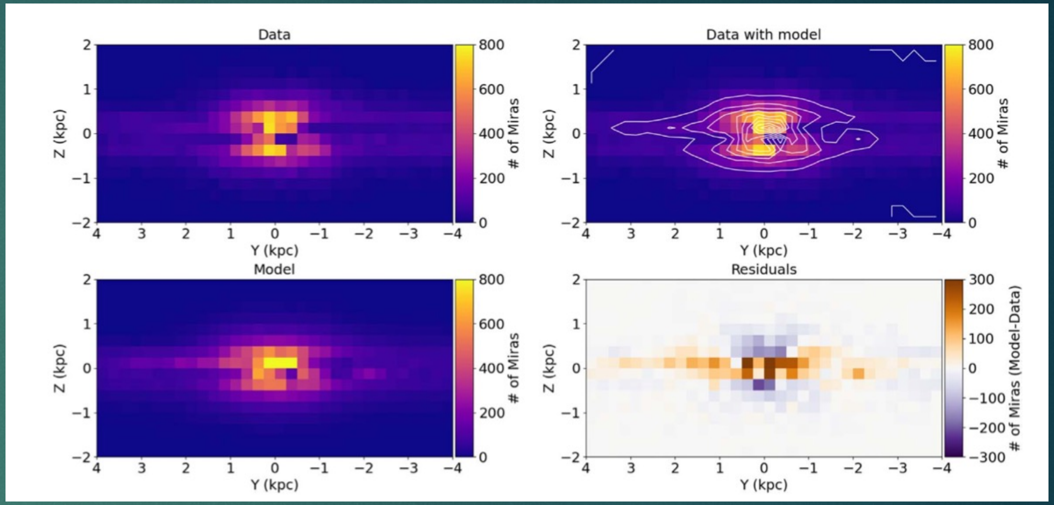
- we generated mock distributions with the inclination of 15 °, 20 °, 25 °, 30 °, 35 °, 40 °, 45 °, 50 ° and 55 °, and we fitted model to these distributions
- Recovered parameters:

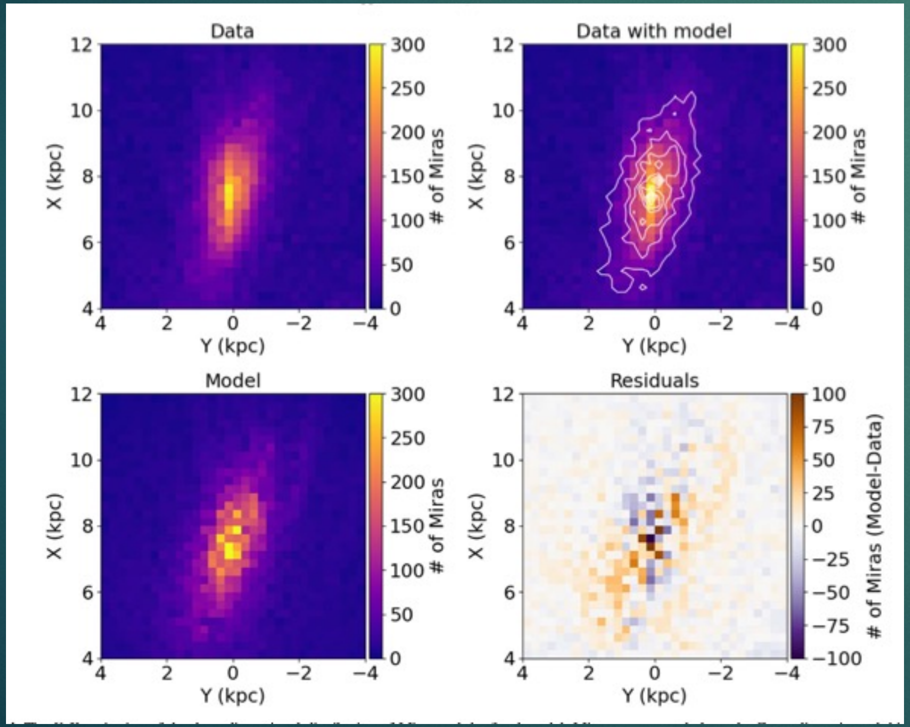


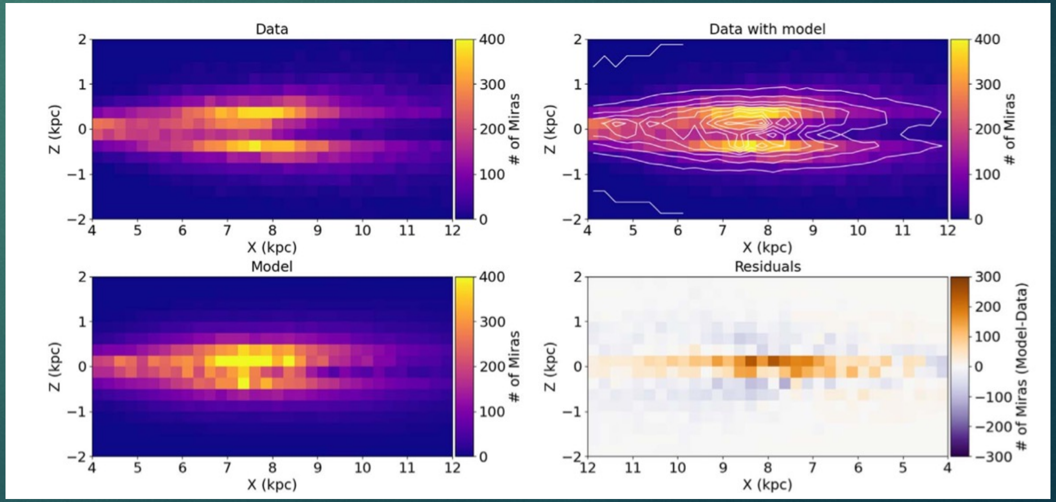
$$\theta_{\rm MCMC} = 0.792(\pm 0.012) \times \theta_{\rm TRUE} - 3.731(\pm 0.448).$$

Final fit

- first step – 44-parameter fit
- $\boldsymbol{\theta}_{\mathrm{MCMC}} = 12.3°$ → from the relation $\boldsymbol{\theta}_{\mathrm{TRUE}} = 20.2°$
- fixed $\boldsymbol{\theta}$ and 43-parameter fit

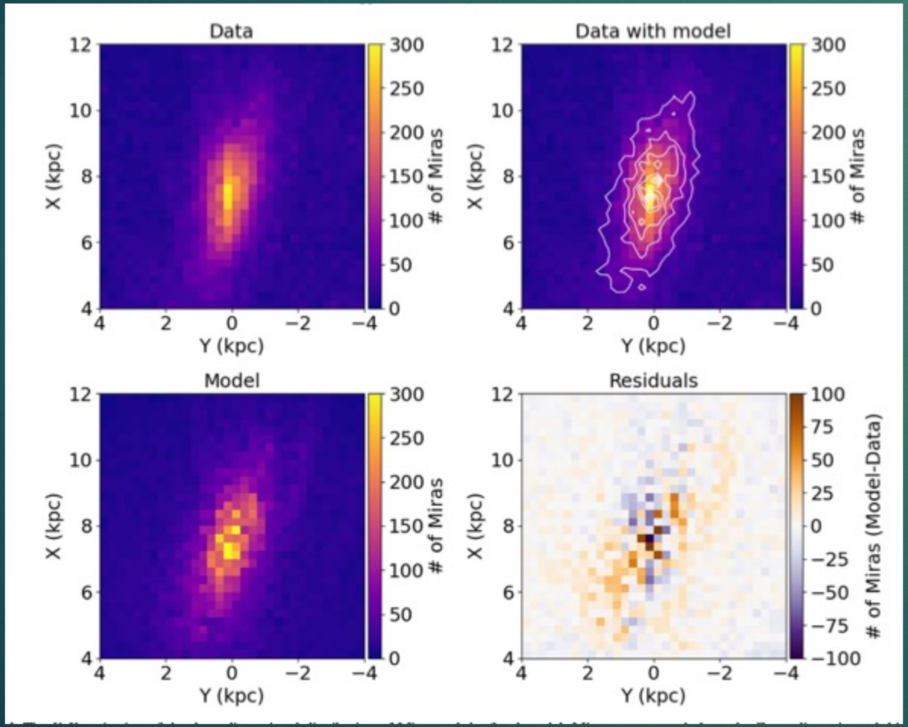


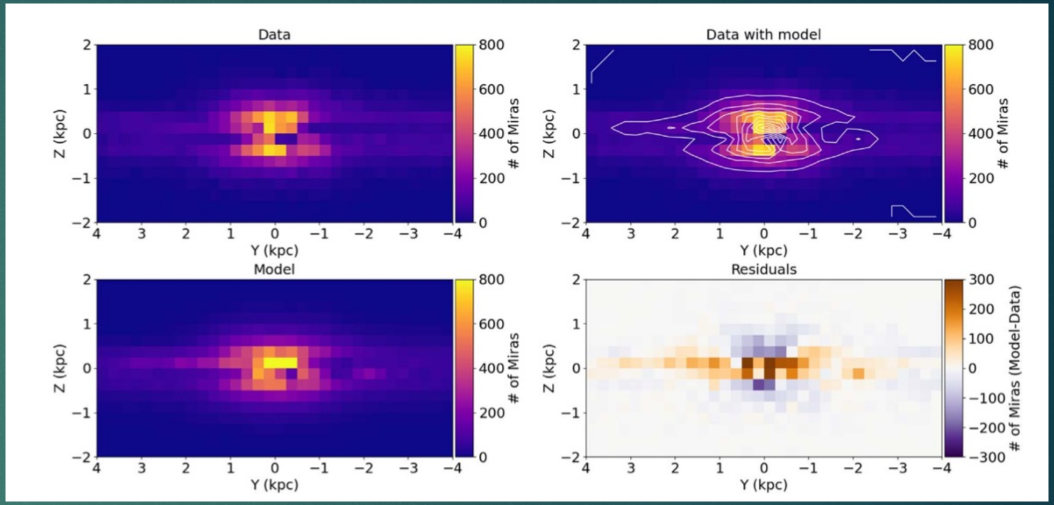


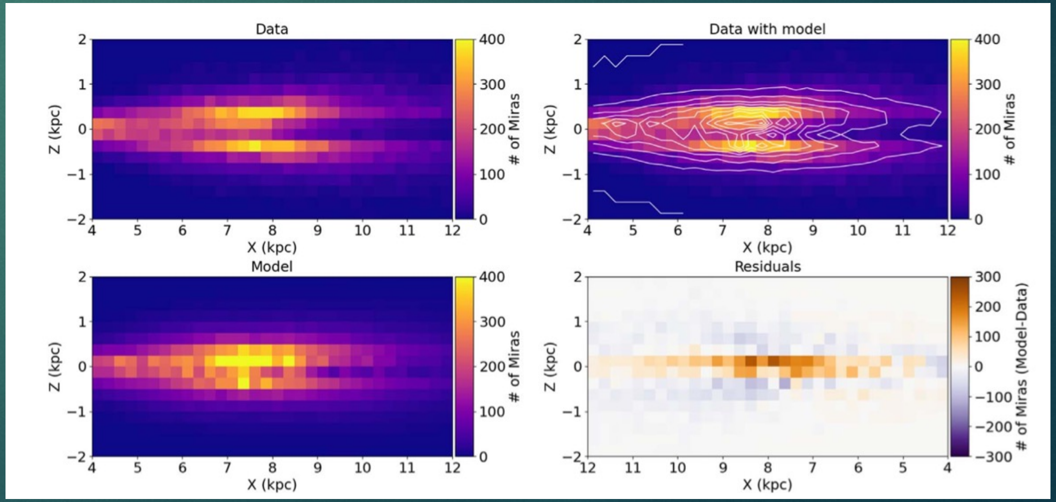


Results

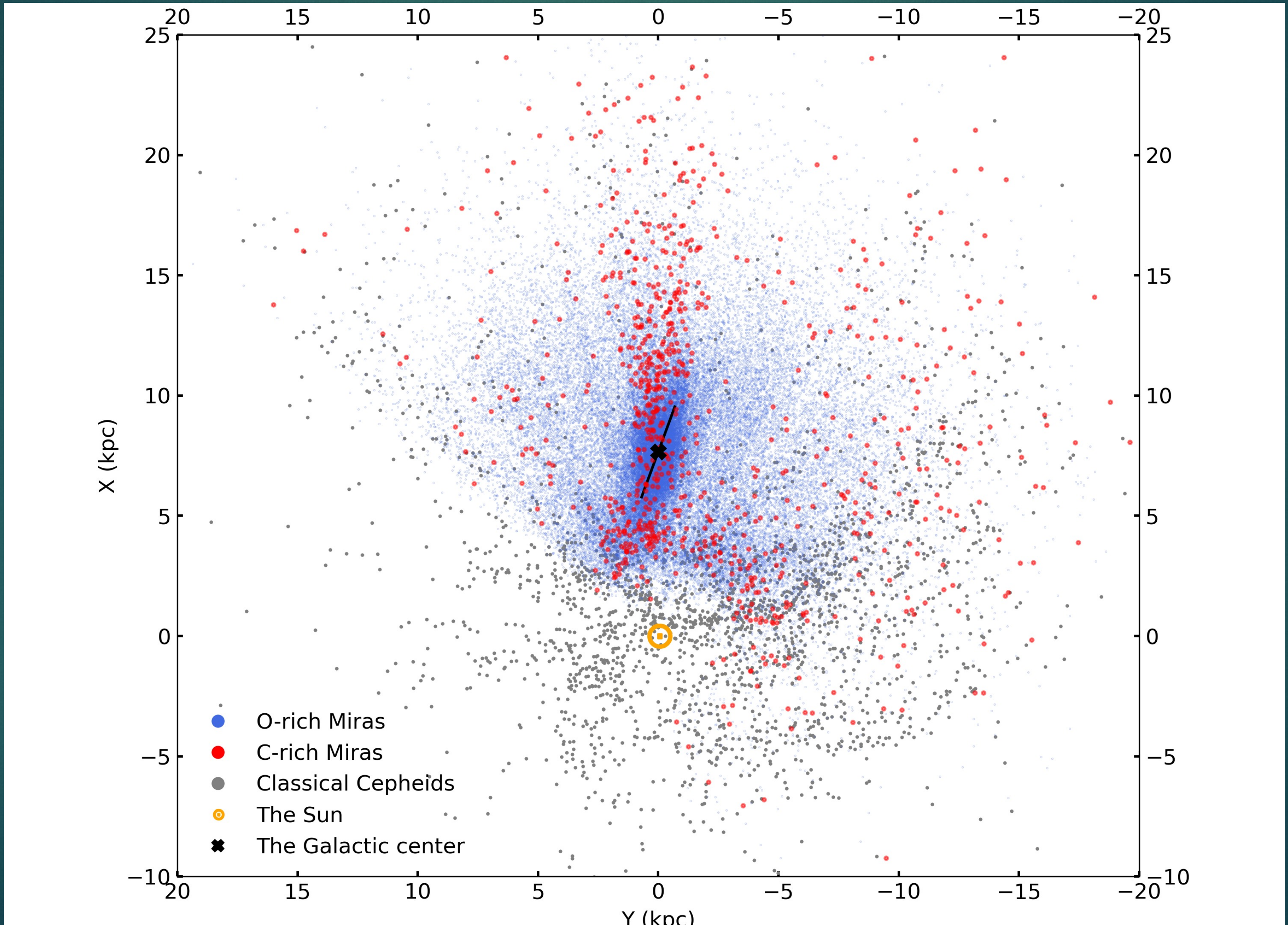
- X-shaped structure exists
- $R_0 = 7.66 \pm 0.01(stat.) \pm 0.39\ (sys.)\ kpc$
- $\theta = 20.2^\circ \pm 0.6^\circ\ (stat.) \pm 0.7^\circ\ (sys.)$



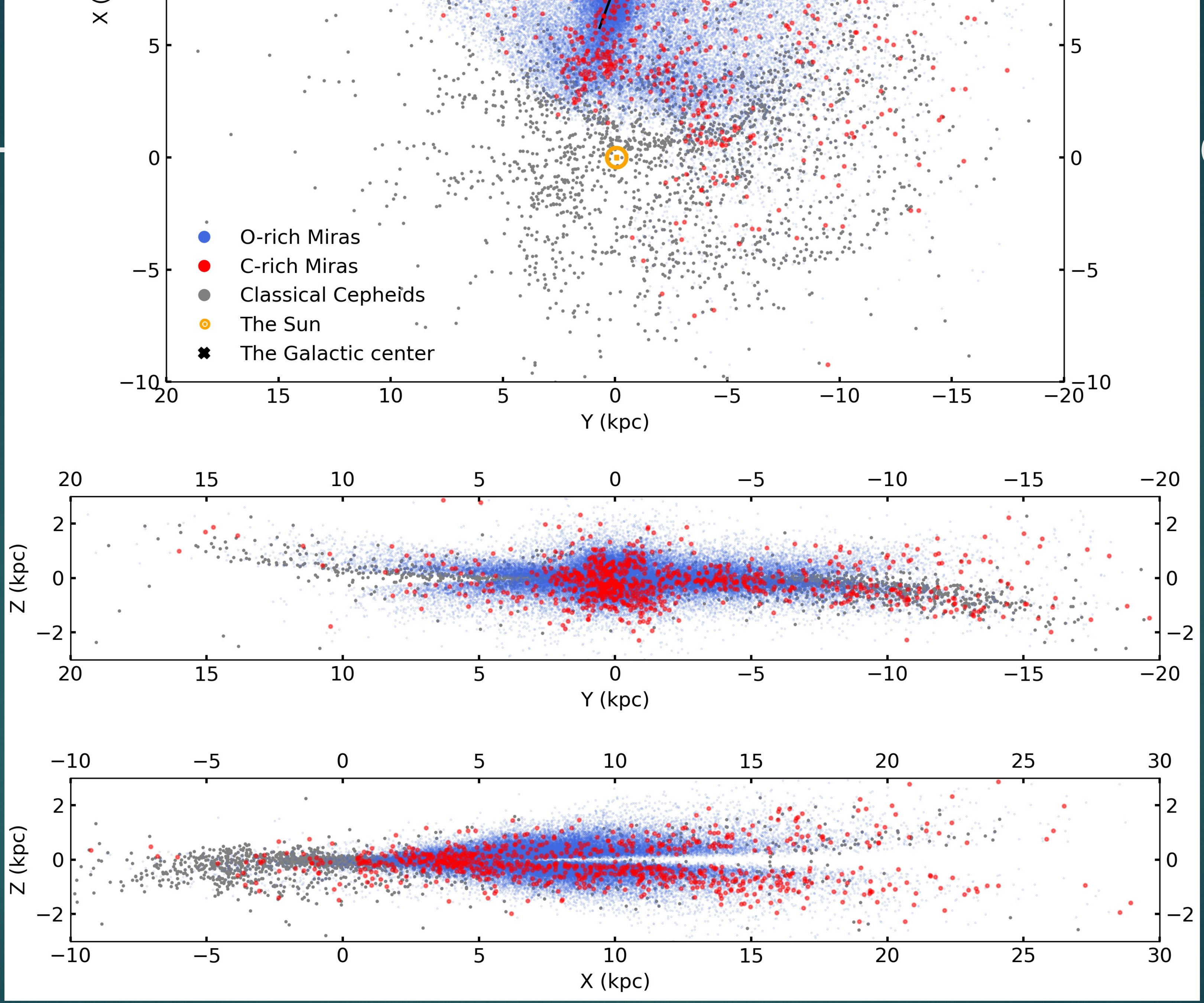




Three-dimensional map of the Milky Way



Three- ay


O-rich Miras
C-rich Miras
Classical Cepheids
The Sun
The Galactic center
Y (kpc)
Z (kpc)
Y (kpc)
Z (kpc)
X (kpc)

Thank you for your attention!