Gravitational instability

- Gravitational instability

Hydrodynamics

If the matter moves in a coordinate system its time evolution can be properly followed by the “matter derivative”:

$$\frac{D}{dt} \equiv \frac{\partial}{\partial t} + \vec{v} \cdot \nabla$$

(It is not enough to know changes of any matter parameter in a point of space, it is also necessary to take into account the change of the position of the matter element in question).

The equations of the classic (non-relativistic) hydrodynamics read:

$$\begin{aligned} \frac{D\vec{v}}{dt} &= -\frac{\nabla P}{\rho} - \nabla\Phi \quad &&\text{(Euler)} \\ \frac{D\rho}{dt} &= -\rho\nabla\cdot\vec{v} &&\text{(continuity)} \\ \nabla^2\Phi &= 4\pi G\rho &&\text{(Poisson)} \\ P &= P(\rho) &&\text{(eq. of state)} \end{aligned}$$

$$\begin{aligned} \rho &= \rho_0 + \delta\rho \equiv \rho_0(1+\delta) \\ \vec{v} &= \vec{v}_0 + \delta\vec{v} \equiv \frac{\dot{a}}{a}\vec{r} + \delta\vec{v} \\ \Phi &= \Phi_0 + \delta\Phi \\ P &= P_0 + \delta P \equiv P_0 + c_S^2\delta\rho \equiv P_0 + \rho_0 c_S^2 \delta \end{aligned}$$

The variables are expanded to the 1st order. The velocity is given as the uniform expansion part plus the peculiar motion. Pressure perturbations are adiabatic since all energy transport phenomena are neglected.

Hydrodynamics

After linearizing equations we see that matter derivative simplifies to:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla_{\vec{r}} \approx \frac{\partial}{\partial t} + \frac{\dot{a}}{a}\vec{r} \cdot \nabla_{\vec{r}} = \frac{d}{dt}$$

The continuity equation gives:

$$\left(\frac{\partial}{\partial t} + \frac{\dot{a}}{a} r^j \nabla_j\right) \delta\rho = -\,\delta\rho\nabla\left(\frac{\dot{a}}{a}\vec{r}\right) - \rho_0 \nabla\delta\vec{v}$$

$$\frac{1}{\rho_0}\frac{d}{dt}\delta\rho = -\,3\frac{\dot{a}}{a}\frac{\delta\rho}{\rho_0} - \nabla\delta\vec{v}$$

$$\frac{1}{\rho_0}\frac{d}{dt}(\rho_0\delta) + 3\frac{\dot{a}}{a}\delta = -\,\nabla\delta\vec{v}$$

$$\frac{d}{dt}\delta = -\nabla\delta\vec{v}$$

Euler eq., its divergence:

$$\left(\frac{\partial}{\partial t} + \frac{\dot{a}}{a} r^j \nabla_j\right) \delta v^i + \frac{\dot{a}}{a}\delta v^i = -\,\frac{\nabla_i \delta P}{\rho_0} - \nabla_i \delta\Phi$$

$$\frac{d}{dt}(\nabla\delta\vec{v}) + 2\frac{\dot{a}}{a}\nabla\delta\vec{v} = -\,c_S^2\nabla^2\delta - \nabla^2\delta\Phi$$

$$\ddot{\delta} + 2\frac{a}{a}\dot{\delta} = +\,c_S^2\nabla^2\delta + 4\pi G\rho_0\delta$$

And finally, for a single plane wave we get the equation describing the evolution of small relative density perturbations. The are written down in a coordinate system comoving with the unperturbed Universe.

$$\delta = \delta_{\vec{k}}\ \exp(i\vec{k}\vec{x}) \qquad \vec{r} = a\vec{x} \qquad \lambda = \frac{2\pi a}{|\vec{k}|}$$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left(\frac{k^2 c_S^2}{a^2} - 4\pi G\rho_0\right)\delta = 0$$

Relativistic hydrodynamics

Bianchi identity implies vanishing of the LHS of the EE → $\nabla_b T^{ab} = 0$
Calculating the divergence we get:

$$D/Ds \equiv u^b \nabla_b.$$

$$T^{ab} = (\epsilon + P) u^a u^b - P g^{ab} \qquad \nabla_b T^{ab} = 0$$

$$u^a \frac{D}{Ds}(\epsilon + P) + u^a (\epsilon + P) \nabla_b u^b + (\epsilon + P) a^a - \nabla^a P = 0$$

"a" is the acceleration. Since 4-velocity is normalized its derivative ("a") must be perpendicular and multiplying by 4-velocity we obtain the energy conservation equation:

$$\frac{D}{Ds}\epsilon + (\epsilon + P) \nabla_b u^b = 0$$

In a small region the Hubble expansion is non-relativistic, so locally matter derivative acting on perturbed variables does not include peculiar velocities and the energy equation becomes:

$$\frac{d}{dt}\delta\epsilon + 3\frac{\dot{a}}{a}(\delta\epsilon + \delta P) + (\epsilon + P)\nabla \delta\vec{v} = 0$$

We examine $\frac{d}{dt}\left(\frac{\delta\epsilon}{\epsilon + P}\right)$

finally obtaining
(Notes)

$$\frac{d}{dt}\left(\frac{\delta\epsilon}{\epsilon + P}\right) = -\nabla \delta\vec{v}$$

Relativistic hydrodynamics

Again we use the slow motion approximation replacing:

$$-\nabla_i \delta\Phi \longrightarrow \delta g_i \qquad \rho \longrightarrow (\epsilon + P)/c^2$$

And using eq.of motion in the form

$$\left(\frac{\partial}{\partial t} + \frac{\dot{a}}{a} r^j \nabla_j\right) \delta v^i + \frac{\dot{a}}{a} \delta v^i = -\frac{c^2 \nabla_i \delta P}{\epsilon + P} + \delta g^i$$

Its divergence:

$$\frac{d}{dt}(\nabla \delta \vec{v}) + 2\frac{\dot{a}}{a}\nabla \delta \vec{v} \;=\; -c_S^2 \nabla^2 \frac{\delta\epsilon}{\epsilon + P} - \frac{4\pi G}{c^2}(\delta\epsilon + 3\delta P)$$

$$\frac{d}{dt}\left(-\frac{d\delta}{dt}\right) + 2\frac{\dot{a}}{a}\left(-\frac{d\delta}{dt}\right) = -\, c_S^2 \nabla^2 \delta - 4\pi G \rho_0 \frac{\epsilon + P}{\epsilon}(1 + 3c_S^2/c^2)\delta$$

For a single plane wave:

Delta= $\dfrac{\delta\epsilon}{\epsilon + P}$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left[\frac{k^2 c_S^2}{a^2} - 4\pi G \rho_0 (1 + w)(1 + 3c_S^2/c^2)\right]\delta = 0$$

Up to “relativistic corrections” this is the same perturbation equation as before.

Growth of small perturbations

In the early Universe (before “matter – radiation equality”) matter is relativistic ($w = 1/3 \text{ and } c_S^2 = c^2/3$) so:

$$\delta \equiv \delta\epsilon/(\epsilon + P)$$

$$\ddot{\delta} + 2\frac{\dot{a}}{a}\dot{\delta} + \left[\frac{k^2 c_S^2}{a^2} - \frac{32\pi G}{3c^2}\epsilon\right]\delta = 0$$

$$\epsilon = \frac{3c^2}{32\pi G t^2} \qquad \text{(early: } t \ll t_{eq}\text{)}$$

$$a(t) \propto t^{1/2} \qquad \text{(early: } t \ll t_{eq}\text{)}$$

$$\lambda(t) = \frac{2\pi a(t)}{k}$$

$$\Rightarrow \ddot{\delta} + \frac{1}{t}\dot{\delta} + \left[\frac{c_S^2}{a^2}\left(\frac{2\pi a}{\lambda}\right)^2 - \frac{1}{t^2}\right]\delta = 0$$

For $\lambda \gg 2\pi c_S t$

$$\ddot{\delta} + \frac{1}{t}\dot{\delta} - \frac{1}{t^2}\delta = 0$$

$$\Rightarrow \delta \propto t^1 \propto a^2 \quad \text{or} \quad \delta \propto t^{-1} \propto a^{-2}$$

On *super-horizon* scales one has one power-law growing mode and another fading away. These solutions are valid until the equality. Since the present energy density of photons is ~10000 times lower than matter density, 1+*z*_eq is of the same order.

Growth of small perturbations

After *equality* and before *recombination* the equation of state becomes complicated (see later: numerical results). After the recombination: decoupling, P~0. After *equality* the matter density is close to critical density of the epoch. This holds until recently, when *dark energy* term starts to play any role, so:

$$\frac{\rho_M(z)}{\rho_c(z)} = \frac{\frac{3H_0^2}{8\pi G}\Omega_M(1+z)^3}{\frac{3H_0^2}{8\pi G}\left(\Omega_M(1+z)^3+\Omega_K(1+z)^2+\Omega_\Lambda\right)} \approx 1 - \frac{\Omega_K}{\Omega_M(1+z)} - \frac{\Omega_\Lambda}{\Omega_M(1+z)^3}$$

$$\Rightarrow \quad \rho_M \approx \frac{1}{6\pi G t^2} \qquad a \propto t^{2/3}$$

$$\ddot{\delta} + \frac{4}{3}\frac{1}{t}\dot{\delta} - \frac{2}{3}\frac{1}{t^2}\delta = 0$$

$$\Rightarrow \quad \delta \propto t^{2/3} \propto a \quad \text{or} \quad \delta \propto t^{-1} \propto a^{-3/2}$$

Again, after the recombination we get a simple analytical solution with one power-law growing and one fading away mode of perturbations. Both baryons and dark matter particles behave in the same way (assuming P=0 for both). (To be strict: there may be differences in the initial conditions between the two components.

Of course the above solutions are valid only when *delta*<<1

Perturbations which are super-horizon until *recombination* may always grow, first as a^2, then as a^1. For smaller scales and to describe the period between *equality* an *recombination* one needs a numerical solution.

Growth of small perturbations

The scale factor a(t) is directly related to the redshift and it serves much better as an independent variable as compared to time when investigating the evolution of perturbations. Since we are going to numerically integrate equations over many orders of magnitude in scale factor, we use ln(a) as independent variable:

$$\frac{d}{dt} = \dot{a}\frac{d}{da} \equiv \frac{\dot{a}}{a}\frac{d}{d\ln a}$$

$$\frac{d^2}{dt^2} = \frac{\dot{a}}{a}\frac{d}{d\ln a}\,\frac{\dot{a}}{a}\frac{d}{d\ln a} = \frac{\dot{a}^2}{a^2}\frac{d^2}{(d\ln a)^2} + \left(\frac{\ddot{a}}{a} - \frac{\dot{a}^2}{a^2}\right)\frac{d}{d\ln a}$$

Substituting:

$$\frac{\dot{a}^2}{a^2}\delta'' + \left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right)\delta' + \left[\frac{k^2 c_S^2}{a^2} - 4\pi G\rho(1+w)(1+3c_S^2/c^2)\right]\delta = 0$$

$$\delta'' + \left(1 + \frac{\ddot{a}a}{\dot{a}^2}\right)\delta' + \frac{a^2}{\dot{a}^2}\left[\frac{k^2 c_S^2}{a^2} - 4\pi G\rho(1+w)(1+3c_S^2/c^2)\right]\delta = 0$$

$$\delta'' + w_1\delta' + w_0\delta = 0$$

Coefficients are explicit functions of cosmological model parameters and the redshift factor.

Growth of small perturbations

An approximate solution for an *always growing* mode and new unknown A:

$$f = \frac{a^2}{a + a_{eq}}$$

$$\begin{aligned} \delta &= Af \\ \delta' &= A'f + Af' \\ \delta'' &= A''f + 2A'f' + Af'' \end{aligned}$$

Function f changes by many orders of magnitude. A wchich is a "correction" to our approximation – not necessarily. We hope for better numerical accuracy with our set of variables. After substitutions:

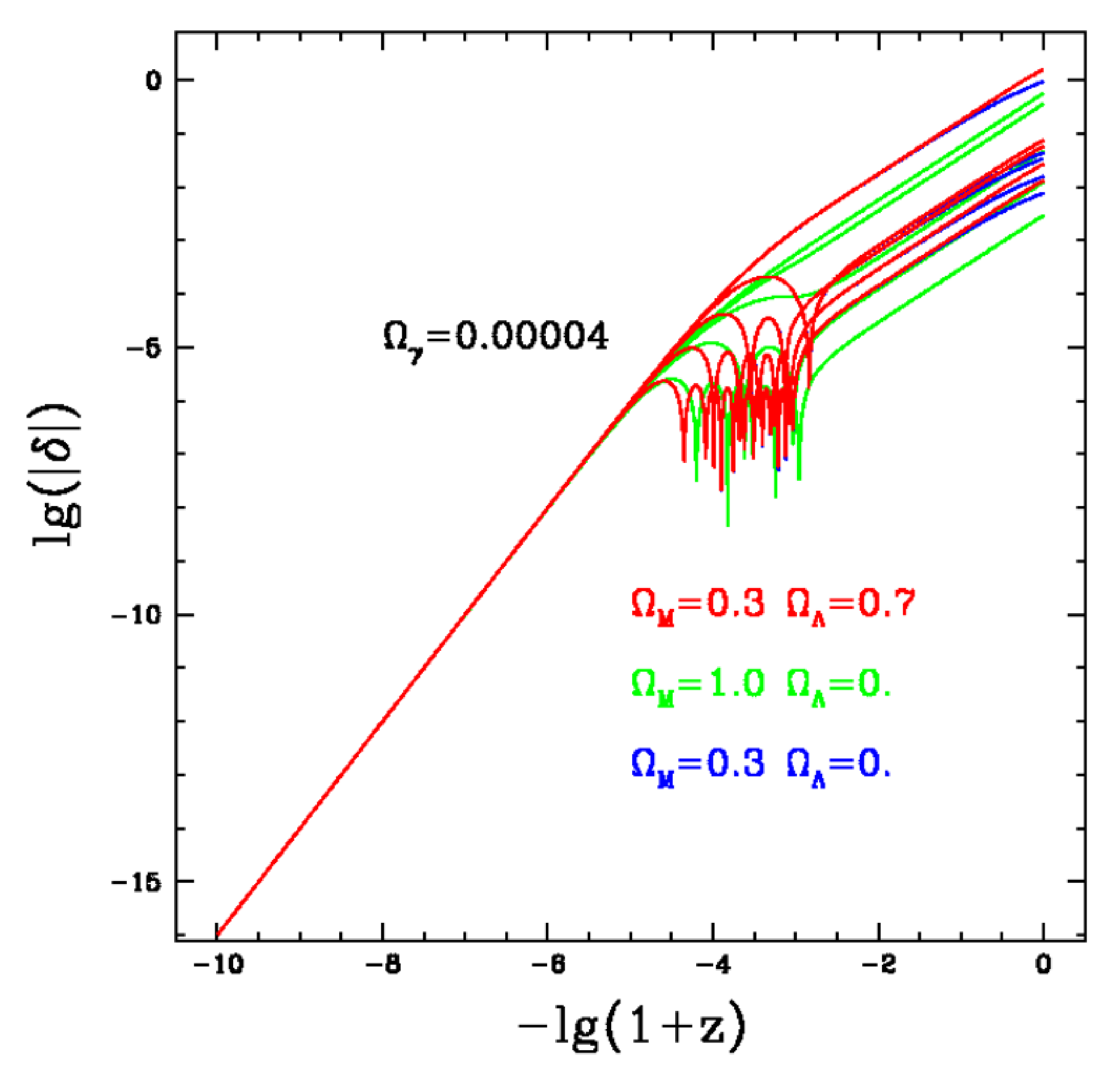
$$(fA'' + 2f'A' + f''A) + w_1(fA' + f'A) + w_0 fA = 0$$

$$A'' + \left(w_1 + 2\frac{f'}{f}\right)A' + \left(w_0 + w_1\frac{f'}{f} + \frac{f''}{f}\right)A = 0$$

With the initial conditions: $A_{\text{init}} = 1 \qquad {A'}_{\text{init}} = 0$

(The initial value is not important – we are only going to compare the total growth between two fixed points ("long before equality" and "today") for different sizes of perturbations. Our approximate solution has good asymptotic behavior, so A should not change at the beginning.)

Growth of small perturbations



Initially growing numerical solutions on a log-log plot. The *super-horizon* modes in the early Universe do not depend on cosmological parameters but the moment they become *sub-horizon* and start to oscillate does. After the *recombination* all grow in the same way, but the level depends on the phase of oscillations at *recombination*. (This example: single fluid approximation).

Condition for instability; Jeans mass

Numerical experience: growth when:

$$[...] < 0 \quad \Rightarrow \quad \lambda > 2\pi c_S t \quad \text{or} \quad \lambda_J = 2\pi c_S t$$

A better parameter: mass (more precisely rest mass, which is conserved; bilion solar masses long ago is bilion today)

$$M_J = \rho_M \lambda_J^3$$

Where "J" stands for Jeans. Jeans mass depends on time. Before *equality* :

$$\rho_M \propto 1/a^3 \qquad \lambda_J \propto t \propto a^2 \qquad M_J \propto a^3 \propto t^{3/2}$$

Between the *equality* and *recombination* it is hard to express.
After the *recombination* photons decouple from baryons and the gas pressure (until recently supported by the radiation pressure) falls dramatically. Jeans mass falls to ~milion solar masses, much less than masses of galaxies.

Always: perturbations which contain more mass than the current Jeans mass may grow and vice versa.

After the *recombination:* practically all perturbations of interest (from the point of structure formation) may grow.

Condition for instability; Jeans mass

Some estimates. At *equality* neutrinos were relativistic (for ~0.2 eV). Their energy density (6 particles, 1 polarization, fermions, lower temperature):

$$\epsilon_\nu = 6 * \frac{1}{2} * \frac{7}{8} * \left(\frac{4}{11}\right)^{4/3} \epsilon_\gamma = 0.681\ \epsilon_\gamma$$

Knowledge of the energy density gives time at *equality*

$$\begin{aligned}
1.68 * \Omega_\gamma * \frac{3H_0^2}{8\pi G} c^2 (1+z_{eq})^4 &= \frac{3c^2}{32\pi G t_{eq}^2} \\
4 * 1.68 * \Omega_\gamma (1+z_{eq})^4 {H_0}^2 &= \frac{1}{t_{eq}^2} \\
t_{eq} = \frac{1}{\sqrt{6.72\Omega_\gamma}} \frac{1}{(1+z_{eq})^2} \frac{1}{H_0} = 6.1 \times 10^{-7} \frac{1}{H_0} &= 8.5 \times 10^3\ \mathrm{y} \\
\lambda_J(t_{eq}) = 2\pi \sqrt{\frac{1}{3}} c t_{eq} &\approx 10\ \mathrm{kpc}
\end{aligned}$$

This is ~10 kpc then corresponding to ~100 Mpc today) →

A huge mass →

$$\begin{aligned}
M_J(t_{eq}) &= \Omega_M \frac{3H_0^2}{8\pi G} (1+z_{eq})^3 * \left(2\pi \sqrt{\frac{1}{3}} c t_{eq}\right)^3 \\
&= \frac{\pi^2}{\sqrt{3}} \frac{\Omega_M}{(6.72\Omega_\gamma)^{3/2}} \frac{1}{(1+z_{eq})^3} \frac{c^2 * (c/H_0)}{G} \approx 3.2 \times 10^{16}\ M_\odot
\end{aligned}$$

Condition for instability; Jeans mass

Similar estimates for *recombination*

$$
\begin{aligned}
\Omega_M * \frac{3H_0^2}{8\pi G}(1+z_{rec})^3 &= \frac{1}{6\pi G t_{rec}^2} \\
\frac{9}{4}\Omega_M (1+z_{rec})^3 {H_0}^2 &= \frac{1}{t_{rec}^2} \\
t_{rec} = \frac{2/3}{\sqrt{\Omega_M(1+z_{rec})^3}}\frac{1}{H_0} = 3.3\times 10^{-5}\frac{1}{H_0} &= 467\times 10^3 \text{ y}
\end{aligned}
$$

Overestimate?
Others get ~380 000 →

The estimate of sound vrlocity at *recombination* is more interesting:

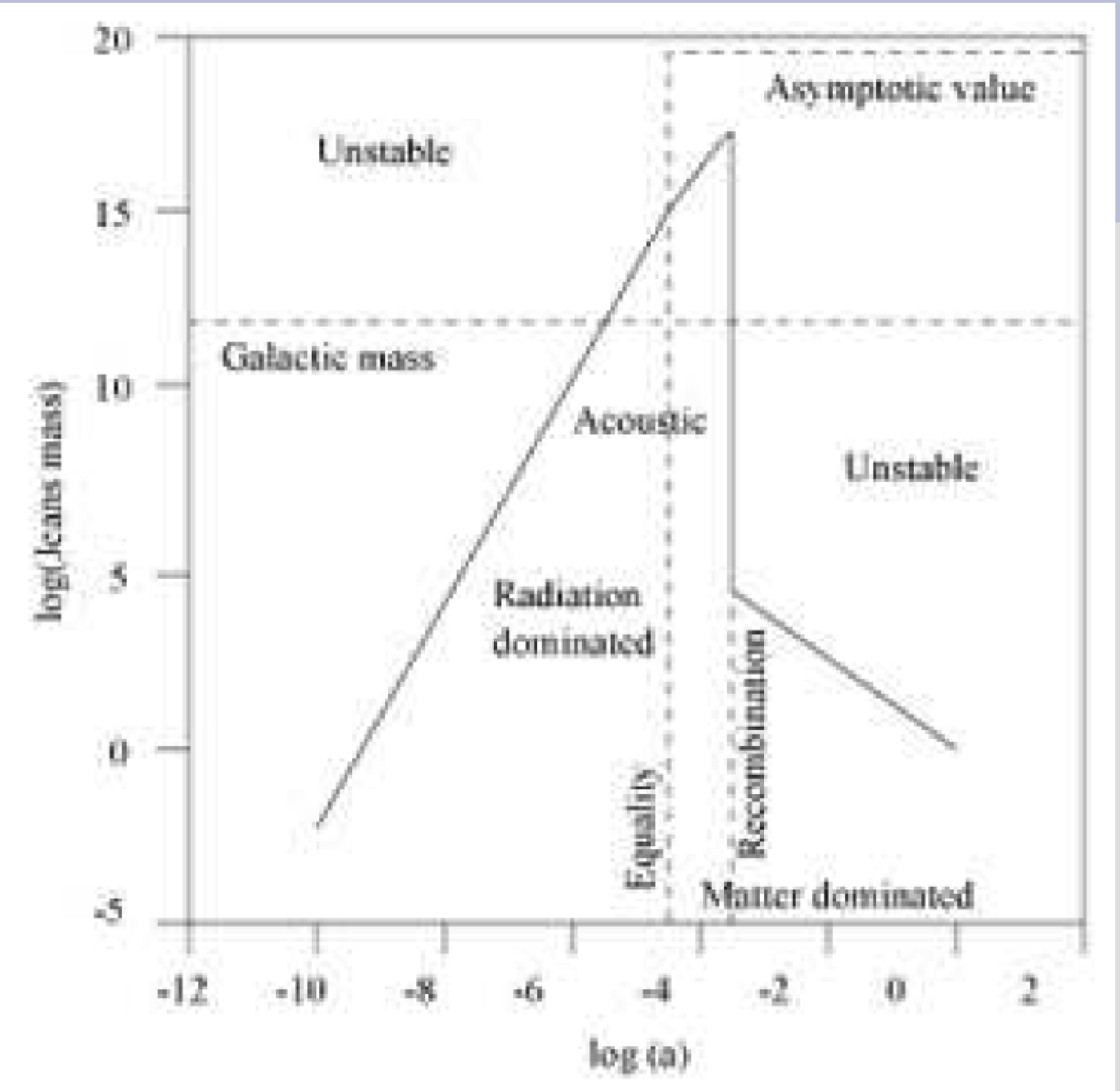
Radiation dominates pressure
Cold matter dominates mass →

$$
\begin{aligned}
P_\gamma &= \frac{1}{3}\Omega_\gamma \frac{3{H_0}^2}{8\pi G}c^2(1+z)^4 \qquad \rho_M = \Omega_M \frac{3{H_0}^2}{8\pi G}(1+z)^3 \\
{c_S}^2 &= \frac{dP_\gamma/dz}{d\rho_M/dz} = \frac{4}{9}c^2\frac{\Omega_\gamma}{\Omega_M}(1+z) \qquad c_S(t_{rec}) \approx 0.25\ c \\
\lambda_J(t_{rec}) &= 2\pi c_S(t_{rec})t_{rec} \approx 225 \text{ kpc} \\
M_J(t_{rec}) &= \Omega_M \frac{3H_0^2}{8\pi G}(1+z_{rec})^3 \lambda_J^3(t_{rec}) \approx 1.3\times 10^{18}\ M_\odot
\end{aligned}
$$

Again huge mass →

With 380 000 y we would get ~7*10^17, still too much?

Condition for instability; Jeans mass



Jeans mass as a function of log(1/(1+z)) . (Source ?) At the *recombination*: ~3*10^17

Our estimate is not so bad...

Transmission factor

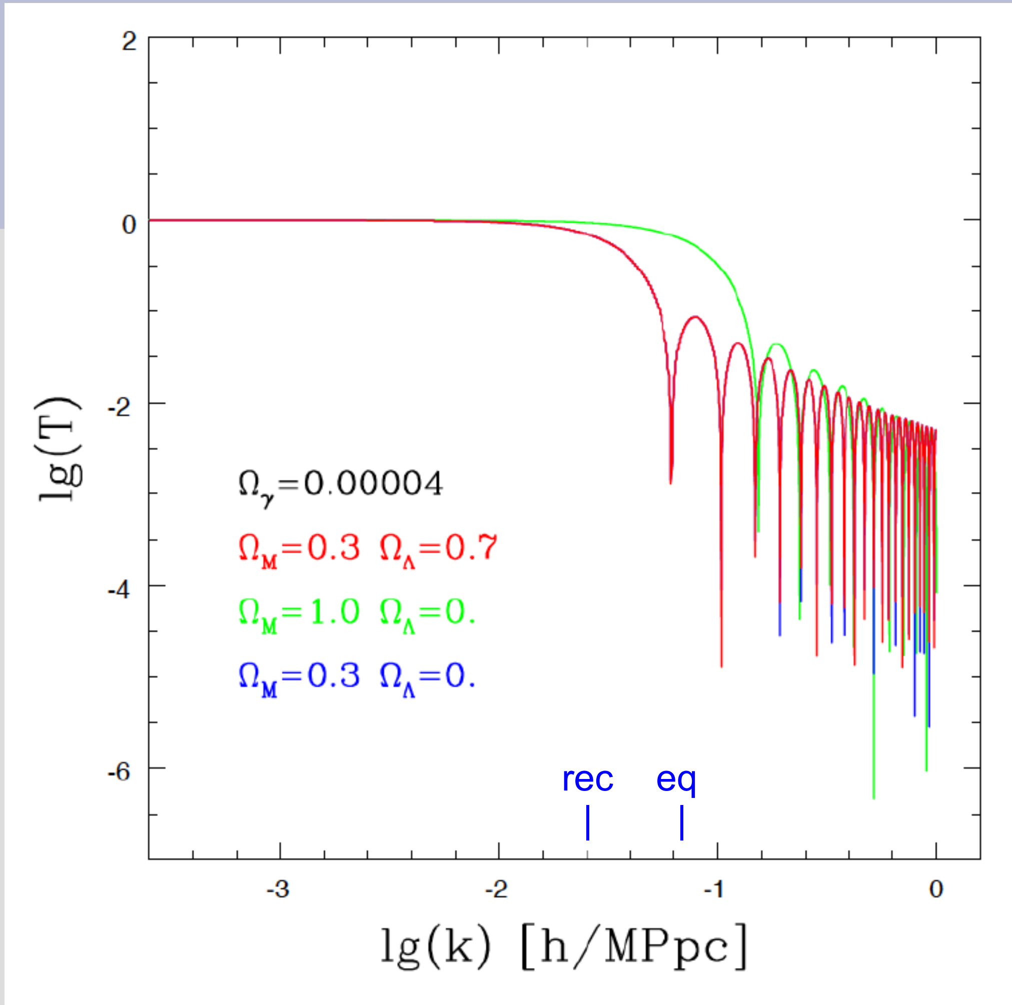
The amplification of a perturbation mode depends on its wave number as seen on previous figure. At the start amplitudes are the same → comparison of the final values gives the relative amplification as compared to an abstract perturbation of infinite size (k=0):

$$Amp(k) = \frac{\delta_k(z=0)}{\delta_k(z=\infty)}$$

$$T(k) = \frac{Amp(k)}{Amp(0)} = \frac{\delta_k(z=0)}{\delta_0(z=0)}$$

T(k) is called “transmission function” “transmission factor” etc and may be used as a characteristic of any amplifier.

Transmission factor

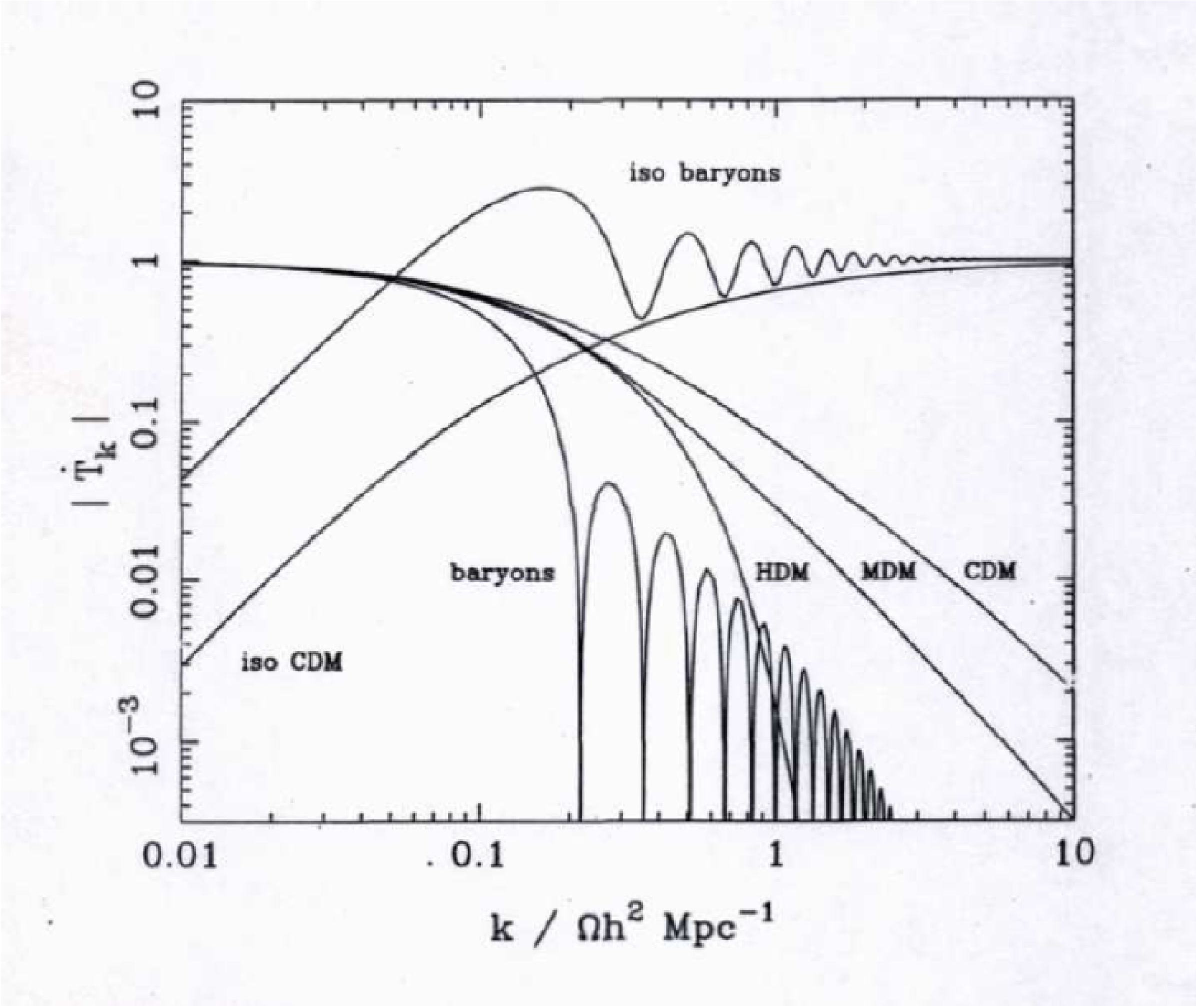


Our previous estimates of Jeans length at *equality* (10 kpc → 100 Mpc today) and *recombination* (225kpc → 247 Mpc) Correspond to lg(k)=-1.2, lg(k)=-1.6 respectively

The transmission factor as a function of a wave number for a single fluid approach. (Qualitative illustration: our calculations are ~OK up to *equality* but then all matter are baryons and the behavior of cold dark matter between *equality* and *recombination* is not well represented)

Important: the characteristic oscillatory modulation of short waves is true, but not exact

Transmission factor



The transmission factor as a function of a wave number for a single fluids of different kinds (baryons, hot, warm, and cold dark matter) Hot dark matter particles (neutrinos?) may travel far and damp perturbations up to large scale. Cold dark matter leaves much more of the small scale structure and is suitable for down – top scenarios, where larger objects are formed by the agglomeration of smaller ones. [Source?]

Two fluid instability
Simplified

We use our equation for single fluid instability. Fluids are not interacting directly but only “through gravitation”. The term reads:

$$\frac{4\pi G}{c^2}(\delta\epsilon + 3\delta P) = \frac{4\pi G}{c^2}\left((\epsilon_\gamma + \epsilon_B)(1+3c_B^2)(1+w_B)\delta_B + \epsilon_X(1+3c_X^2)(1+w_X)\delta_X\right)$$

So it is the same as before but from two sources. The pressure to energy density ratio and the sound velocity (in *c* units) are:

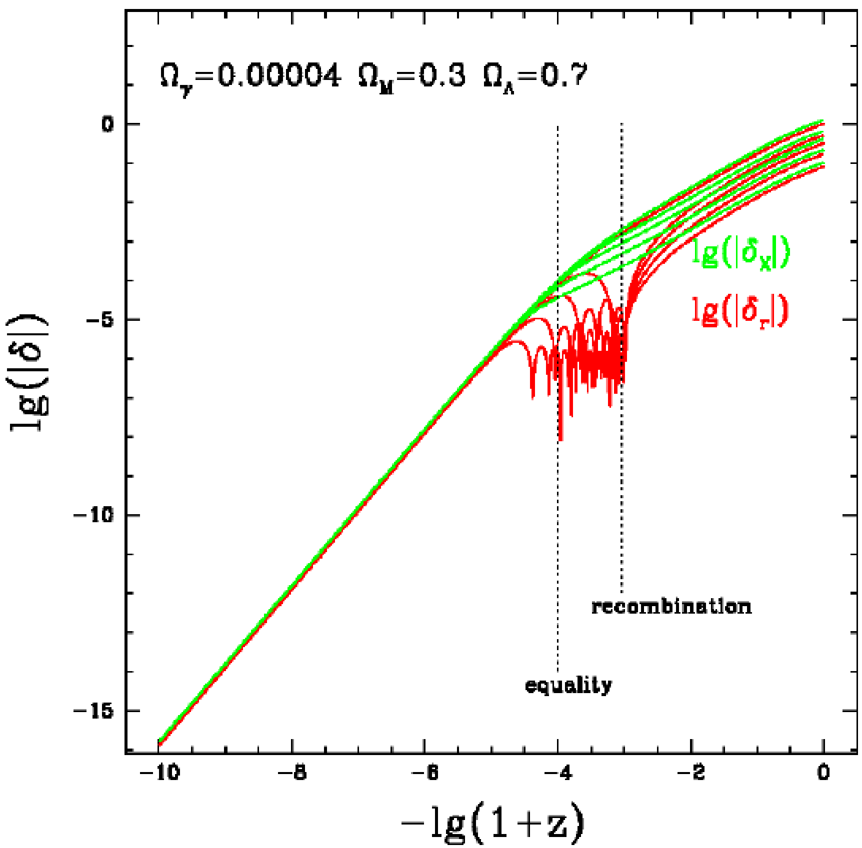
$$\begin{aligned} w_B &= \frac{P_\gamma}{\epsilon_\gamma + \epsilon_B} = \frac{\frac{1}{3}\Omega_\gamma(1+z)^4}{\Omega_\gamma(1+z)^4 + \Omega_B * (1+z)^3} = \frac{1}{3}\frac{\Omega_\gamma(1+z)}{\Omega_\gamma(1+z)+\Omega_B} \\ c_B^2 &= \frac{dP/dz}{d\epsilon/dz} = \frac{1}{3}\frac{4\Omega_\gamma(1+z)}{4\Omega_\gamma(1+z)+3\Omega_B} \end{aligned}$$

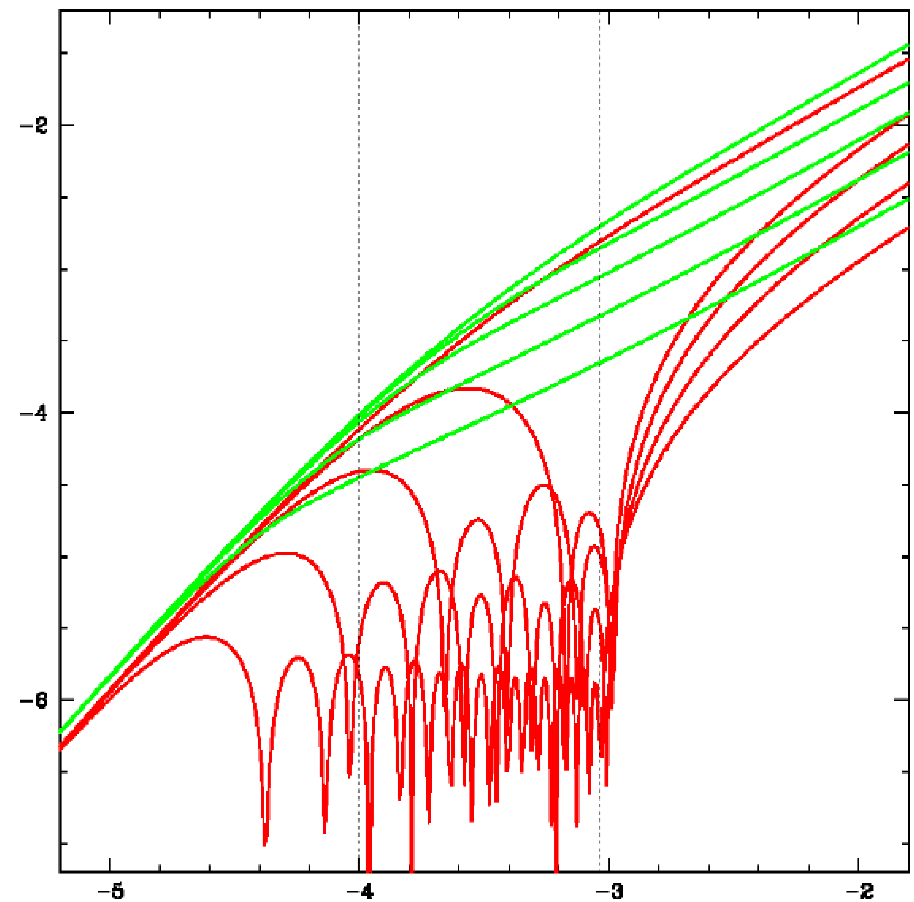
And for dark matter both vanish. Our set of equations:

$$\begin{aligned} \ddot{\delta}_B + 2\frac{\dot{a}}{a}\dot{\delta}_B &+ \left[\frac{k^2 c_B^2 c^2}{a^2}\delta_B - \frac{4\pi G}{c^2}\left((\epsilon_\gamma + \epsilon_B)(1+3c_B^2)(1+w_B)\delta_B + \epsilon_X(1+3c_X^2)(1+w_X)\delta_X\right)\right] = 0 \\ \ddot{\delta}_X + 2\frac{\dot{a}}{a}\dot{\delta}_X &+ \left[\frac{k^2 c_X^2 c^2}{a^2}\delta_X - \frac{4\pi G}{c^2}\left((\epsilon_\gamma + \epsilon_B)(1+3c_B^2)(1+w_B)\delta_B + \epsilon_X(1+3c_X^2)(1+w_X)\delta_X\right)\right] = 0 \end{aligned}$$

It may be applied to any two fluids not directly interacting, with different sound velocities. For dark matter we use c_X=w_X=0. Using the trick delta_B=A_B*f, delta_x=A_X*f and logarithmic independent variable we get the solutions.

Two fluid instability

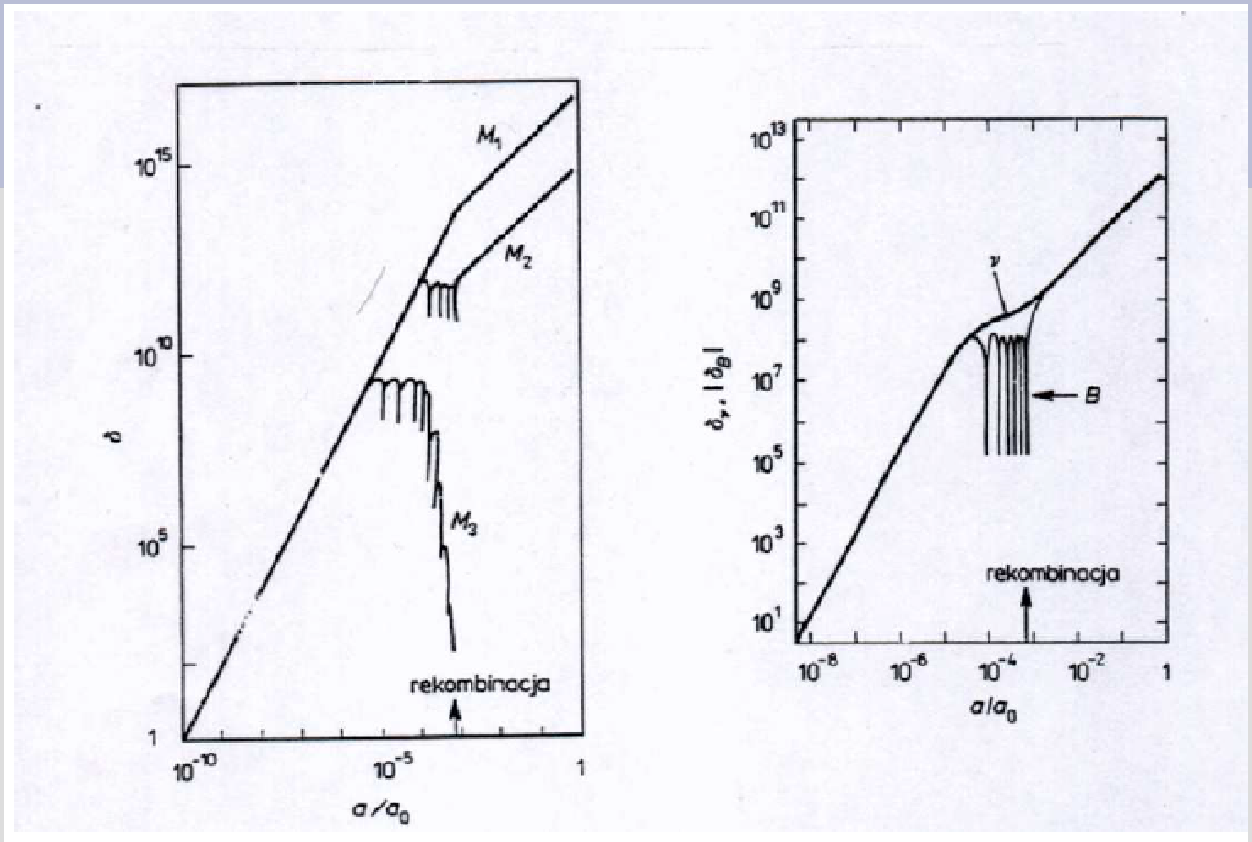




Instability of two fluids (before *equality* relativistic gas + cold dark matter, after *recombination* cold baryons + cold dark matter)
Now OK.

(Fast “improvements” to an old single-fluid program)

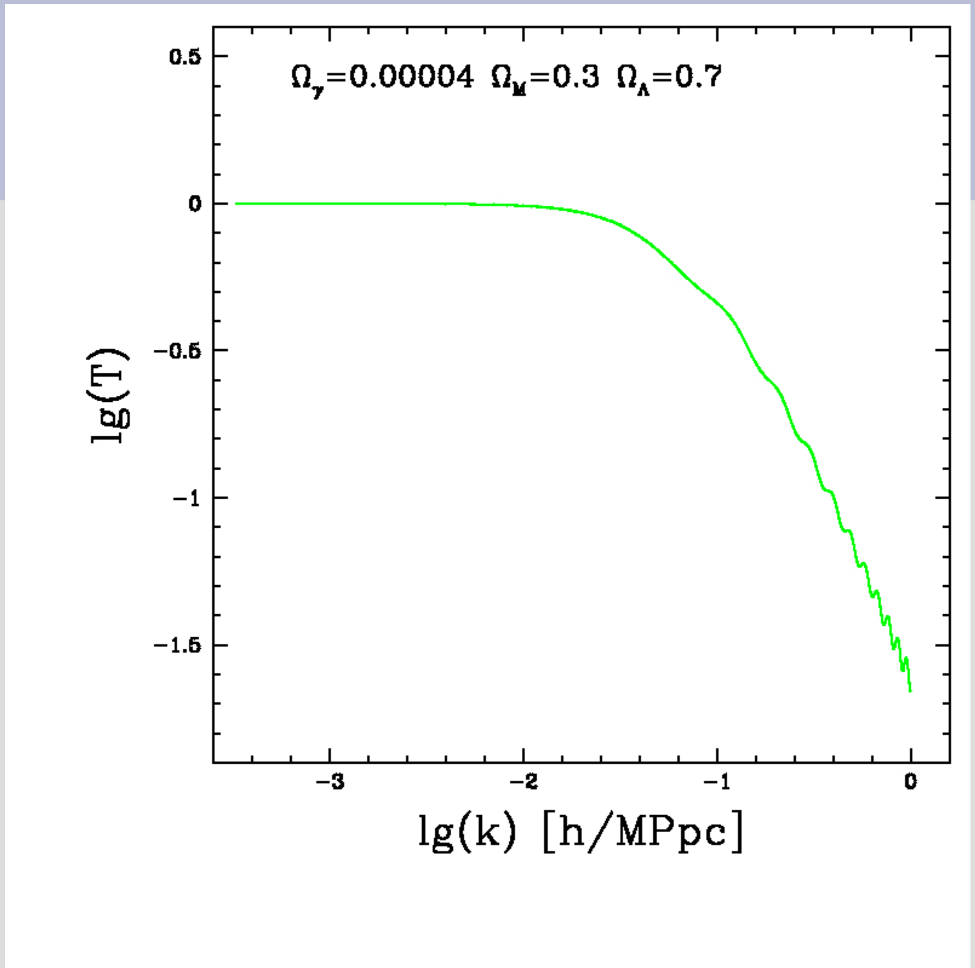
One and two fluid instability



Left: instability of one fluid. Perturbations of different mass: M_1>max(M_J) always growing. M_2: intermediate. M_3<~10^10 M_sun: Silk effect, damping of oscillations when photon free path becomes of the order of perturbation size.
Right: two fluids (radiation plasma ← → baryons plus massive neutrinos)

Very old. (Now out of fashion ← used neutrinos of ~10 eV rest energy)

Two fluid instability



Instability of two fluids: transmission factor. Small scales (large wave numbers) are “filtered out” (“low pass filter” - not very sharp)

Preliminary. Qualitatively good. Not precise.

Spectrum of fluctuations

Any function (with some conditions) can be expressed as Fourier transform:

$$\delta(\vec{x}) = \int d_3 k \delta_{\vec{k}} \exp(-i\vec{k}\vec{x})$$

Where *delta* describes relative fluctuations of matter density. Average fluctuation is zero, but average square is not:

$$\frac{1}{V}\int_V d_3x|\delta^2(\vec{x})| =$$
$$\frac{1}{V}\int_V d_3x \int d_3k'\delta_{\vec{k}'} \exp(-i\vec{k}'\vec{x}) \int d_3k''\delta^*_{\vec{k}''} \exp(+i\vec{k}''\vec{x})$$
$$\rightarrow \int d_3k'\delta_{\vec{k}'} \int d_3k''\delta^*_{\vec{k}''}\delta_{\mathrm{Dirac}}(\vec{k}' - \vec{k}'')$$
$$= \int d_3k|\delta^2_{\vec{k}}| \approx 4\pi \int \frac{dk}{k} \, k^3|\delta^2_k|$$

Dirac's delta results from space integration at $V \rightarrow$ *infinity*. 3D → 1D integration results from the assumed isotropy: anything can depend on $|k|$ but not on its direction. The function integrated over ln(k) is called *power spectrum of density fluctuations*:

$$P_\delta(k) \equiv k^3|\delta^2(k)|$$

Harrison – Zeldovich spectrum

$$P_\delta(k) \equiv k^3|\delta^2(k)| \qquad P_{\delta\Phi}(k) \equiv k^3|\delta\Phi^2(k)|$$

(Similarly potential fluctuations have some power spectrum). Zeldovich and Harrison postulated independently that the spectra should be of a power-law form (since there is no natural scale in the early Universe and no way to propose more complicated function):

$$|\delta_k^2| \sim k^n$$

$$\triangle\delta\Phi_{\vec{k}} = -\, k^2\delta\Phi_{\vec{k}} = 4\pi G\rho_0\delta_{\vec{k}}$$

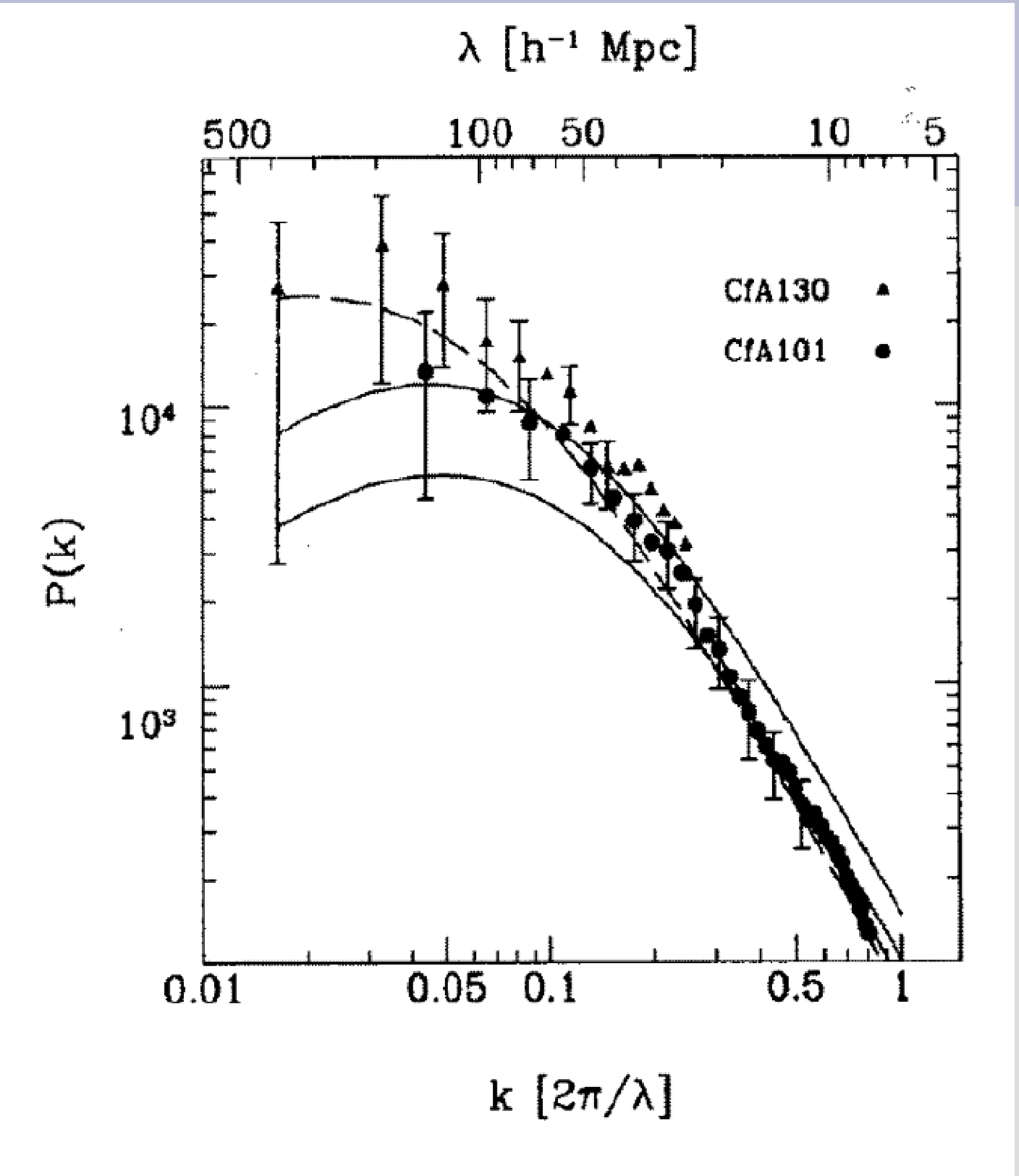
$$P_{\delta\Phi}(k) \sim k^3\frac{|\delta_k^2|}{k^4} = \frac{|\delta_k^2|}{k}$$

The Poisson equation gives the relation between density and potential fluctuations which gives the spectrum of potential perturbations. Power law functions become infinite at zero or infinity, depending on the power index. Exception: power index=0. One would not like potential spectrum becoming infinite at small scales, which would mean the presence of many low mass black holes in the Universe. Strong very large scale fluctuation may lead to the gravitational collapse of the Universe as whole. To avoid both extremes one postulates:

$$\frac{|\delta_k^2|}{k} \sim k^0 \qquad |\delta_k^2| \sim k^1 \qquad n = 1$$

Which is the Harrison – Zeldowich initial spectrum of density fluctuations. The amplitude is not given, must be fitted to observations.

Fluctuations spectrum: examples



CfA observations of galaxy distribution in space compared with some theoretical predictions.