## Bayes in the sky: Bayesian inference and model selection in cosmology

by Roberto Trotta

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### Agenda

Act 1: Bayesian inference

Act 2: Cosmology 101

Act 3: Bayesian approach in cosmology

# Act 1 Bayesian Inference

"Bayesians address the question everyone is interested in by using assumptions no-one believes, while frequentists use impeccable logic to deal with an issue of no interest to anyone" - Louis Lyons

### Definition of probability

probability as frequency:

"the number of times the event occurs over the total number of trials, in the limit of an infinite series of equiprobable repetitions." Probability as degree of belief:

"probability is a measure of degree of belief about a proposition"

### Bayes' Theorem

$$p(H|d,I) = \frac{p(d|H,I)p(H|I)}{p(d|I)}.$$

d-observed data

H-hypothesis

I-previously known information

p(d|H,I) - likelihood function

p(H|I) - prior probability

p(d|I) - Bayesian evidence

p(H|d,I) - posterior probability



The only known portrait that is probably of Bayes.

### Prior

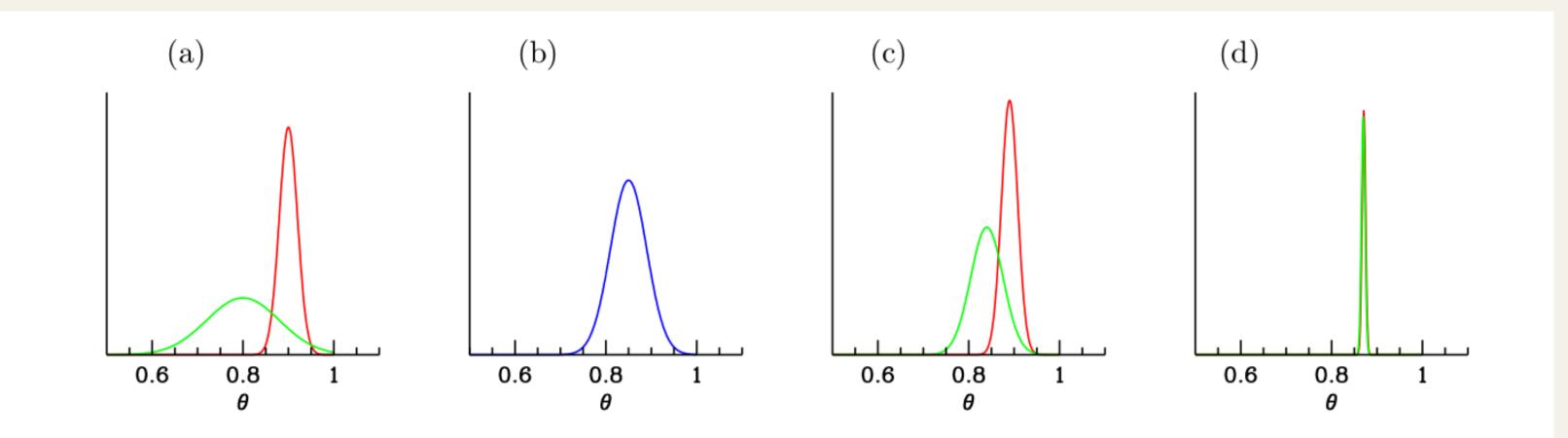


Figure 2. Converging views in Bayesian inference. Two scientists having different prior believes  $p(\theta|I_i)$  about the value of a quantity  $\theta$  (panel (a), red and green pdf's) observe one datum with likelihood  $\mathcal{L}(\theta)$  (panel (b)), after which their posteriors  $p(\theta|m_1)$  (panel (c), obtained via Bayes Theorem, Eq. (8)) represent their updated states of knowledge on the parameter. After observing 100 data points, the two posteriors have become essentially indistinguishable (d).

### Evidence

$$\frac{p(\mathcal{M}_0|d)}{p(\mathcal{M}_1|d)} = B_{01} \frac{p(\mathcal{M}_0)}{p(\mathcal{M}_1)}$$

and the Bayes factor  $B_{01}$  is the ratio of the models' evidences:

$$B_{01} \equiv \frac{p(d|\mathcal{M}_0)}{p(d|\mathcal{M}_1)}$$
 (Bayes factor).

#### Roberto Trotta

Table 1. Empirical scale for evaluating the strength of evidence when comparing two models,  $\mathcal{M}_0$  versus  $\mathcal{M}_1$  (so-called "Jeffreys' scale"). Threshold values are empirically set, and they occur for values of the logarithm of the Bayes factor of  $|\ln B_{01}| = 1.0$ , 2.5 and 5.0. The right-most column gives our convention for denoting the different levels of evidence above these thresholds. The probability column refers to the posterior probability of the favoured model, assuming non-committal priors on the two competing models, i.e.  $p(\mathcal{M}_0) = p(\mathcal{M}_1) = 1/2$  and that the two models exhaust the model space,  $p(\mathcal{M}_0|d) + p(\mathcal{M}_1|d) = 1$ .

$ \ln B_{01} $	Odds	Probability	Strength of evidence
< 1.0 1.0 2.5 5.0	$\begin{array}{l} \lesssim 3:1 \\ \sim 3:1 \\ \sim 12:1 \\ \sim 150:1 \end{array}$	< 0.750 $0.750$ $0.923$ $0.993$	Inconclusive Weak evidence Moderate evidence Strong evidence

### Advantages of the method

- application of Bayes' Theorem recovers frequentist results (in the long run) for cases simple enough where such result exist, while remaining applicable to questions that cannot even be asked in a frequentist context
- Bayesian inference deals effortlessly with nuisance parameters.
- prior information is highly relevant and omitting it would result in seriously wrong inferences
- Bayesian statistics only deals with the data that were actually observed

# Act 2 Cosmology

## "vanilla" ACDM (cold dark matter) cosmological model

The Universe is expanding. The Universe is homogeneous and isotropic.

Described by Friedmann–Robertson–Walker metric:

$$ds^{2} = dt^{2} - \frac{a^{2}(t)}{c^{2}} \left[ \frac{dr^{2}}{1 + \kappa r^{2}} + r^{2} (d\theta^{2} + \sin^{2}\theta d\phi^{2}) \right]$$

The scale factor a(t) describes the expansion of the Universe, and it is related to redshift z by:

$$1 + z = \frac{a(t_0)}{a(t)},$$

The relation between redshift and comoving distance r:

$$a_0 dr = \frac{c}{H_0} \left[ \Omega_{\kappa} (1+z)^2 + \Omega_{\Lambda} + (\Omega_b + \Omega_{cdm})(1+z)^3 + (\Omega_{\gamma} + \Omega_{\nu})(1+z)^4 \right]^{-1/2} dz,$$

Cosmic microwave background

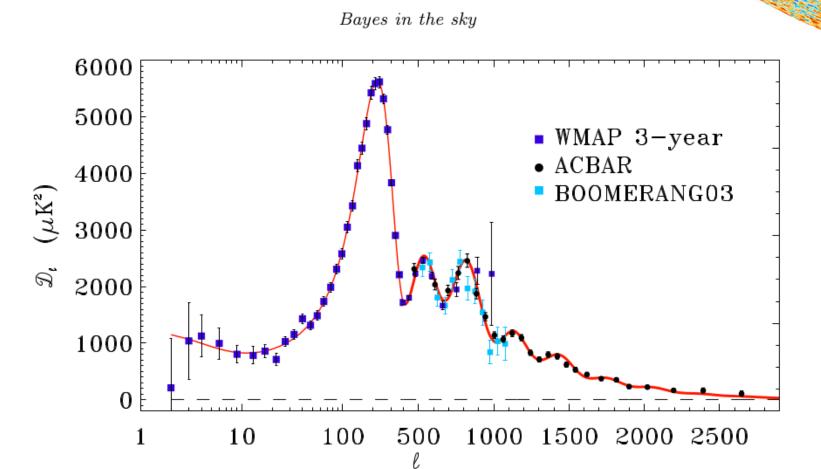


Figure 4. State–of–the-art cosmic microwave background temperature power spectrum measurements along with the best–fit ΛCDM model (solid line), showing data from WMAP 3–yr [80], the Boomerang 2003 flight [101] and ACBAR [97] (from [97]).

#### CREDIT ESA/Planck Collaboration

The CMB is a snapshot of the oldest light in our cosmos, imprinted on the sky when the Universe was just 380 000 years old. It shows tiny temperature fluctuations that correspond to regions of slightly different densities, representing the seeds of all future structure: the stars and galaxies of today.

### Other observables

- Large scale structures
- Weak gravitational lensing
- Supernovae type la

# Act 3 Cosmological Bayes

### Why use the Bayesian approach?

- The complexity of the modelling of both our theories and observations will always increase
- The discovery zone for new physics is when a potentially new effect is seen at the 3–4 σ level. This is when tantalizing suggestion for an effect starts to accumulate but there is no firm evidence yet. In this potential discovery region a careful application of statistics can make the difference between claiming or missing a new discovery.
- A better appreciation of the interpretation of statistical statements might help in identifying robust claims from spurious ones.
- Limited resources mean that we need to focus our efforts on the most promising avenues.
- Sometimes there will be no better data!

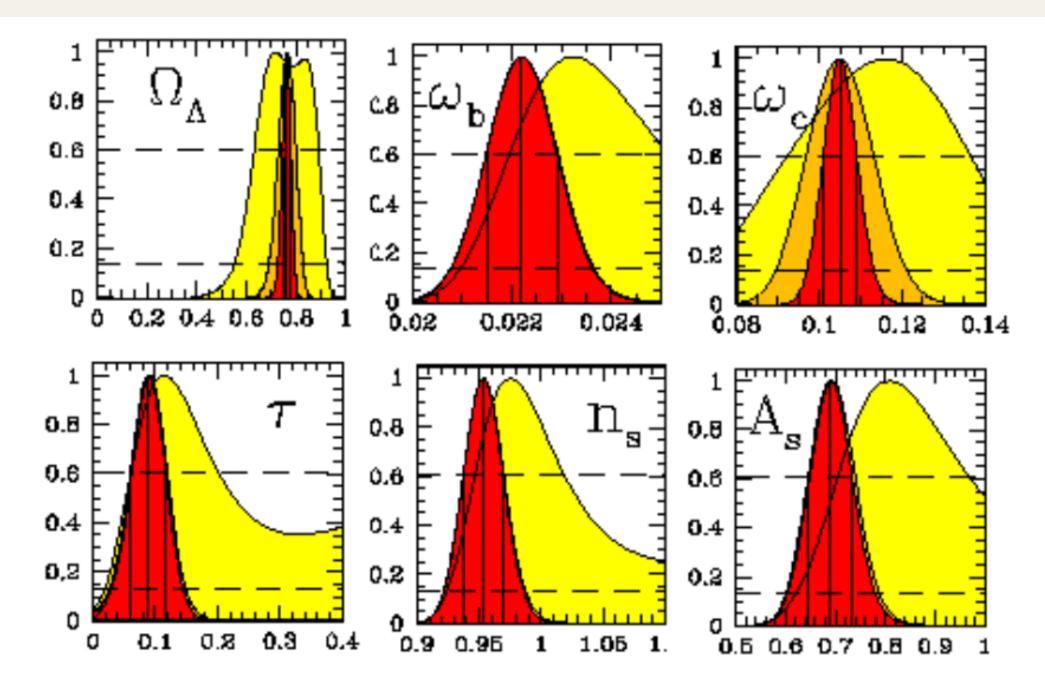


Figure 6. Posterior constraints on key cosmological parameters from recent CMB and large scale structure data, compare Table 3 Top row, from left to right, posterior pdf (normalized to the peak) for the cosmological constant density in units of the critical density, the (physical) baryons and cold dark matter densities. Bottom row, from left to right: optical depth to reionization, scalar tilt and scalar fluctuations amplitude. Yellow using WMAP 1–yr data, orange WMAP 3–yr data and red adding Sloan Digital Sky Survey galaxy distribution data. Spatial flatness and adiabatic initial conditions have been assumed. This set of only 6 parameters (plus 2 other nuisance parameters not shown here) appear currently sufficient to describe most cosmological observations (adapted from [105]).

#### Parameters

Table 3. State-of-the art cosmological parameter inference from WMAP 3-year CMB data [80] and Sloan Digital Sky Survey data [105]. Posterior median and 68% posterior region, obtained for flat priors on the parameter set in the top section, with the exception of the reionization optical depth  $\tau$ , for which a flat prior has been adopted on  $\exp(-2\tau)$  instead (adapted from [105]).

Parameter	Value	Meaning	Definition
Matter bu	dget parameters		
$\Theta_s$ $\omega_b$ $\omega_c$	$0.5918^{+0.0020}_{-0.0020} \ 0.0222^{+0.0007}_{-0.0007} \ 0.1050^{+0.0041}_{-0.0040} \  ext{ditions parameters}$	CMB acoustic angular scale fit (degrees) Baryon density Cold dark matter density	$\begin{split} \Theta_s &= r_s(z_{\rm rec})/d_A(z_{\rm rec}) \times 180/\pi \\ \omega_b &= \Omega_b h^2 \approx \rho_b/(1.88 \times 10^{-26} \rm kg/m^3) \\ \omega_c &= \Omega_{\rm cdm} h^2 \approx \rho_c/(1.88 \times 10^{-26} \rm kg/m^3) \end{split}$
$A_s$ $n_s$	0.690 <sup>+0.045</sup> 0.953 <sup>+0.016</sup> on history (abrupt reionizat	Scalar fluctuation amplitude Scalar spectral index tion)	Primordial scalar power at $k=0.05/{\rm Mpc}$ Primordial spectral index at $k=0.05/{\rm Mpc}$
τ Nuisance i	$0.087^{+0.028}_{-0.030}$ parameters (for galaxy powers)	Reionization optical depth	
$b$ $Q_{ m nl}$	1.896 <sup>+0.074</sup> <sub>-0.069</sub> 30.3 <sup>+4.4</sup> <sub>-4.1</sub>	Galaxy bias factor  Nonlinear correction parameter	See [105] for details. See [105] for details.
Derived pa	arameters (functions of thos	se above)	
$egin{array}{l} \Omega_{ m tot} \ h \ \Omega_b \ \Omega_c \end{array}$	1.00 (flat Universe assumed) $0.730^{+0.019}_{-0.019}$ $0.0416^{+0.0019}_{-0.0018}$ $0.197^{+0.016}_{-0.015}$	Total density/critical density Hubble parameter Baryon density/critical density CDM density/critical density	$ \Omega_{\text{tot}} = \Omega_m + \Omega_{\Lambda} = 1 - \Omega_{\kappa}  h = \sqrt{(\omega_b + \omega_c)/(\Omega_{\text{tot}} - \Omega_{\Lambda})}  \Omega_b = \omega_b/h^2  \Omega_{\text{cdm}} = \omega_c/h^2 $
$\Omega_m$ $\Omega_{\Lambda}$ $\sigma_8$	$0.239_{-0.017}^{+0.018}$ $0.761_{-0.018}^{+0.017}$ $0.756_{-0.035}^{+0.035}$	Matter density/critical density Cosmological constant density/critical density Density fluctuation amplitude	$\Omega_m = \Omega_b + \Omega_{\rm cdm}$ $\Omega_{\Lambda} \approx h^{-2} \rho_{\Lambda} (1.88 \times 10^{-26} \text{kg/m}^3)$ See [105] for details.

### A Bayesian Perspective on Evidence for

### Evolving Dark Energy

Dily Duan Yi Ong, David Yallup and Will Handley

Goal: comparison of dynamic dark energy model (ω0, ωa CDM) and cosmological constant model (ΛCDM) Data:

- measurements from Dark Energy Spectroscopic Instrument (DESI) DR2 Baryon Acoustic Oscillation (BAO)
- Planck 2018 CMB
- Type la supernovae catalogs Pantheon, Union3, DES-Y5;

Methodology: Bayesian inference

	This Work (Bayesian)		DESI Collab. (Frequentist)	
Dataset	$-\ln B$	Significance	$\Delta \chi^2_{ m MAP}$	Significance
Individual Datasets				
DESI DR2	$-1.47 \pm 0.11$	n/a	-4.7	$1.7\sigma$
DESI DR1	$-1.64 \pm 0.10$	n/a	_	_
Pairwise Combinations				
DESI DR2 + CMB  (no lensing)	$-0.38 \pm 0.25$	n/a	-9.7	$2.7\sigma$
DESI DR1 + CMB  (no lensing)	$-0.50 \pm 0.25$	n/a		
DESI DR2 + CMB	$-0.57 \pm 0.26$	n/a	-12.5	$3.1\sigma$
DESI DR1 + CMB	$-0.38 \pm 0.26$	n/a		
DESI DR2 + Pantheon+	$-2.77 \pm 0.12$	n/a	-4.9	$1.7\sigma$
DESI DR1 + Pantheon+	$-2.98 \pm 0.11$	n/a		
DESI DR2 $+$ Union3	$+0.25\pm0.12$	$1.39 \pm 0.31 \sigma$	-10.1	$2.7\sigma$
DESI DR1 $+$ Union3	$+0.42\pm0.11$	$1.59 \pm 0.10  \sigma$	_	_
DESI DR2 + DES-Y5	$+1.56\pm0.12$	$2.33\pm0.06\sigma$	-13.6	$3.3\sigma$
$DESI\ DR1\ +\ DES-Y5$	$+0.84 \pm 0.11$	$1.92\pm$ 0.07 $\sigma$	—	_
Triplet Combinations				
DESI DR2 + CMB + Pantheon+	$-1.70 \pm 0.26$	n/a	-10.7	$2.8\sigma$
DESI DR2 + CMB + Union3	$+1.37\pm0.27$	$2.23 \pm 0.15 \sigma$	-17.4	$3.8\sigma$
$DESI\ DR2 + CMB + DES-Y5$	$+3.32 \pm 0.27$	$3.07\pm$ 0.10 $\sigma$	-21.0	$4.2\sigma$

Table I. Comparison of Bayesian and frequentist model comparison for  $w_0w_a\text{CDM}$  vs  $\Lambda\text{CDM}$ . DESI results from Table VI of Ref. [1]. Negative  $\ln B$  favours  $\Lambda\text{CDM}$ ; negative  $\Delta\chi^2_{\text{MAP}}$  favours  $w_0w_a\text{CDM}$ . Bayesian significances are only computed when  $\ln B > 0$  (favouring  $w_0w_a\text{CDM}$ ); n/a indicates cases where the Bayes factor favours  $\Lambda\text{CDM}$ .

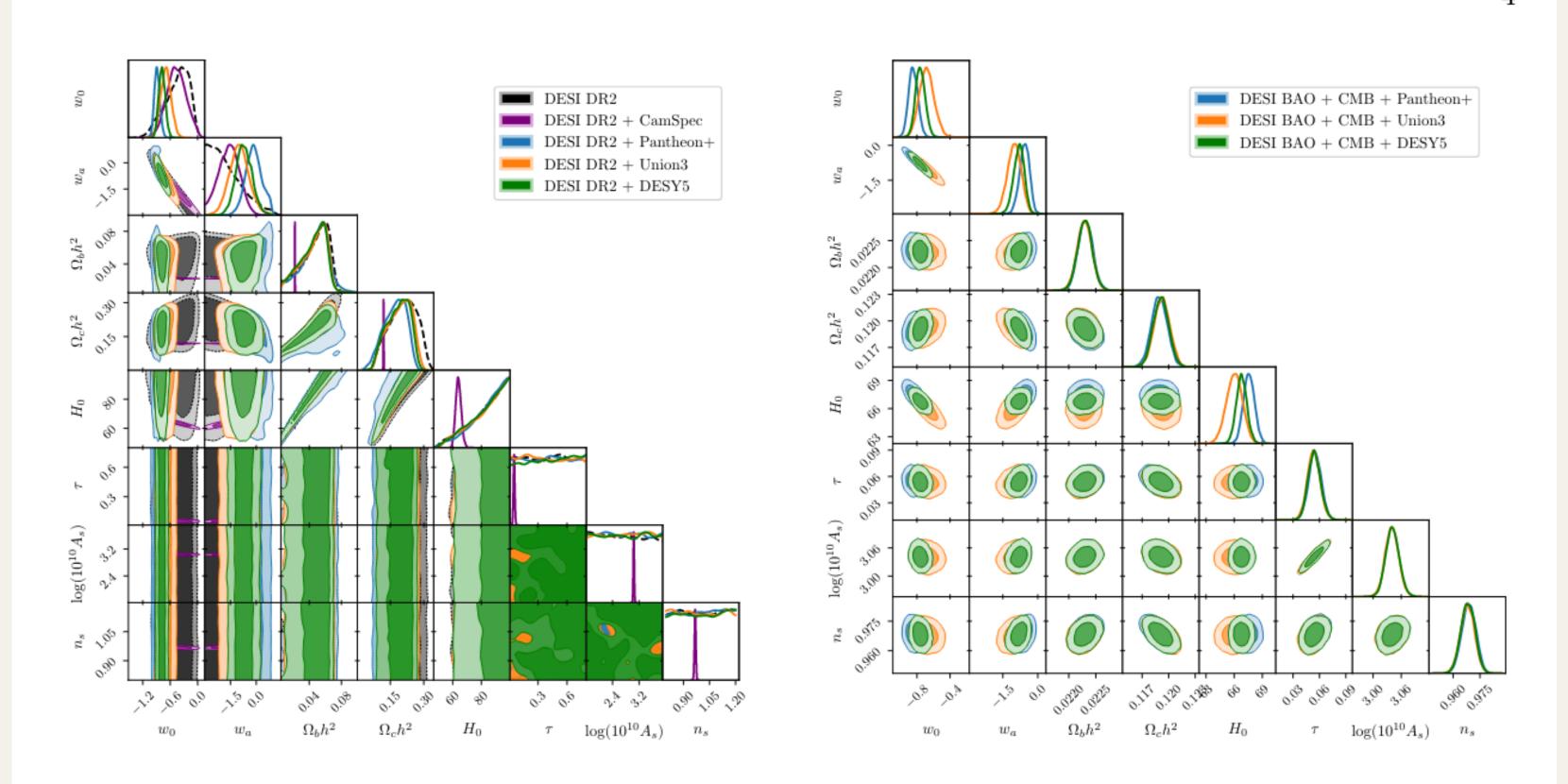


Figure 1. Posterior comparisons in  $w_0w_a$ CDM showing the full cosmological parameter space. Left: DESI DR2 alone (black dashed) and pairwise combinations with CMB (purple), Pantheon+ (blue), Union3 (orange), and DES-Y5 (green). Right: Triplet combinations with DESI BAO + CMB combined with Pantheon+ (blue), Union3 (orange), and DES-Y5 (green). The differing constraints on  $w_0$  and  $w_a$  reflect the varying levels of tension between DESI BAO and each additional dataset. Figures produced with anesthetic [26].

### Empirical validation: Investigating the $\Lambda_s$ CDM model with new DESI BAO observations

Manish Yaday Archana Dixit Anirudh Pradhan M S Barak

Goal: examining AsCDM

Data:

- DESI BAO data
- Pantheon SNIa sample
- full Planck 2018 data;

They investigate how the free parameter ( $z^{\dagger}$  - sign-switching cosmological constant) of  $\Lambda$ sCDM affects (or does not) other cosmological parameters in given datasets.

Results: z<sup>†</sup> remains largely unconstrained. Future large-scale surveys such as Euclid, DESI, and the Roman Space Telescope are expected to improve this situation by providing higher-precision measurements across a broader redshift range.

Howwever, given the current uncertainties and model dependencies, we cannot definitively claim that  $\Lambda$ sCDM, or any dynamical dark energy model, is favored over the standard cosmological constant.

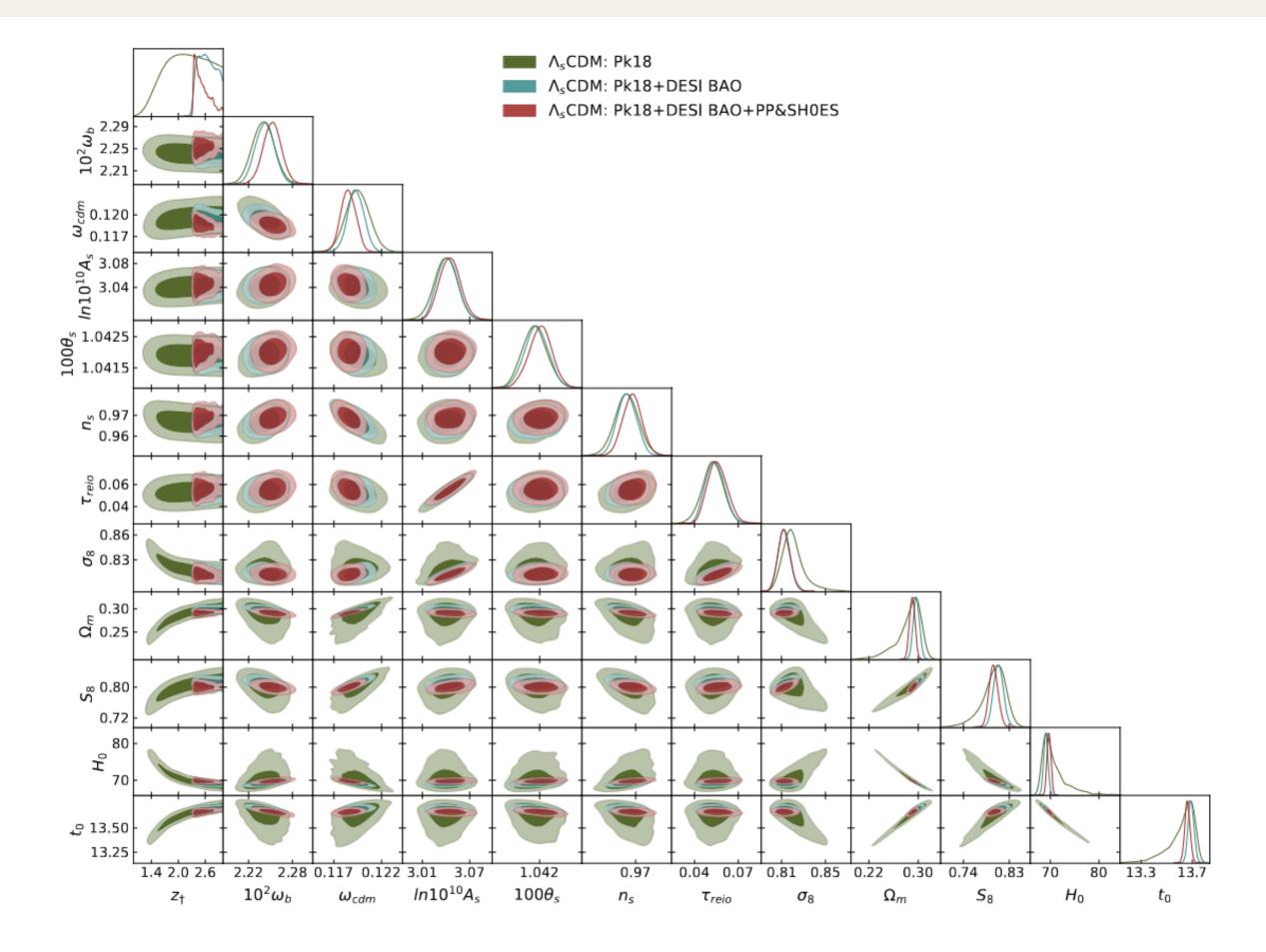


FIG. 1. Marginalized one- and two-dimensional distributions (68% and 95% CLs) of the  $\Lambda_s$ CDM model parameters for different datasets combinations: Pk18, Pk18+ DESI BAO, and Pk18+ DESI BAO+PP&SH0ES.

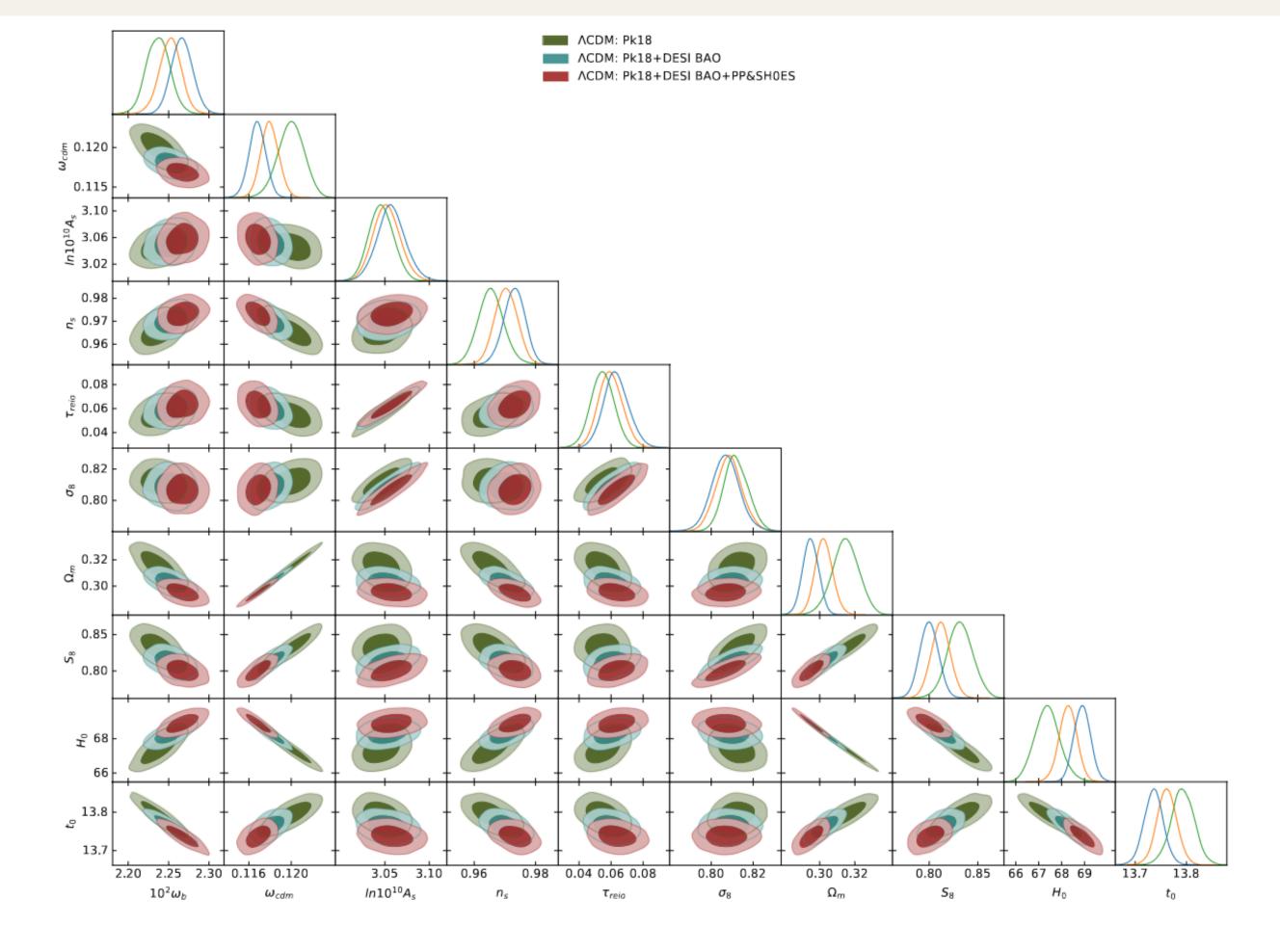


FIG. 2. Marginalized one- and two-dimensional distributions (68% and 95% CLs) of the  $\Lambda$ CDM model parameters for different datasets combinations: Pk18, Pk18+ DESI BAO, and Pk18+ DESI BAO+PP&SH0ES.

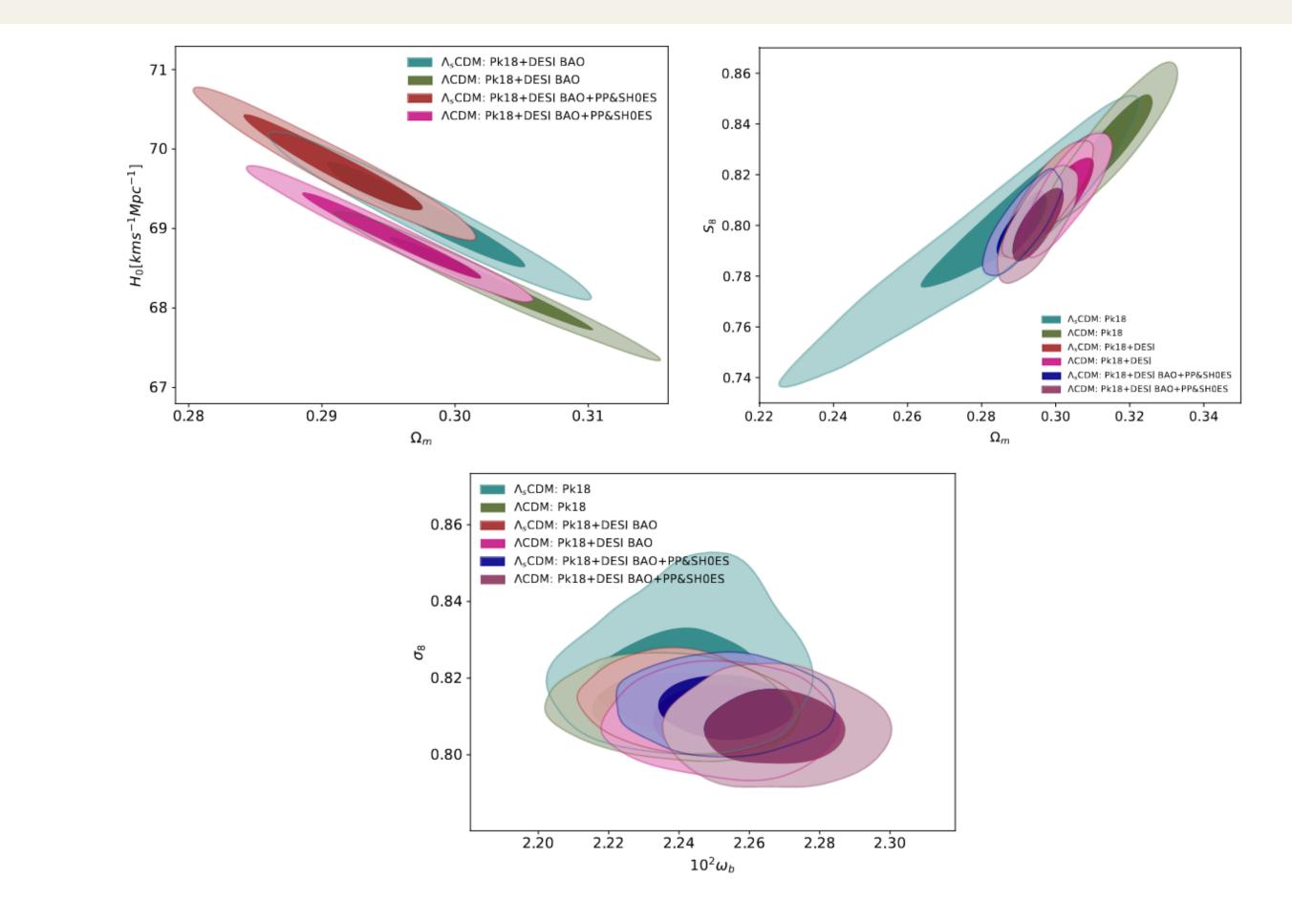


FIG. 3. The left side 2D contour plots in the  $H_0$ - $\Omega_{\rm m}$  plane, and the right side 2D contour plots in the  $S_8$ - $\Omega_{\rm m}$  plane, and the middle bottem 2D contour plots in the  $sigma_8$ - $10^2\omega_{\rm b}$  plane shown at 68% and 95% CL for the  $\Lambda_{\rm s}$ CDM and  $\Lambda$ CDM models with Pk18, Pk18+DESI BAO, and Pk18+DESI BAO+PP&SH0ES.

### References:

- <a href="https://en.wikipedia.org/wiki/Thomas\_Bayes">https://en.wikipedia.org/wiki/Thomas\_Bayes</a> (Figure slide 5)
- Introduction to Cosmology Barbara Ryden
- A Bayesian Perspective on Evidence for Evolving Dark Energy https://doi.org/10.48550/arXiv.2511.10631
- DESI DR2 Results II: Measurements of Baryon Acoustic Oscillations and Cosmological Constraints <a href="https://doi.org/10.48550/arXiv.2503.14738">https://doi.org/10.48550/arXiv.2503.14738</a>
- Empirical validation: Investigating the  $\Lambda_s$ CDM model with new DESI BAO observations <a href="https://doi.org/10.48550/arXiv.2509.26049">https://doi.org/10.48550/arXiv.2509.26049</a>