

DIFFERENTIAL EVOLUTION OPTIMIZATION ALGORITHM

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Statistics Journal Club Apr 20
'lies, damned lies and statistics'

AGENDA

- bio inspired computing techniques;
- evolutionary algorithms;
- differential evolution;
- how does it work?
- Parameter tuning;
- Mutation and recombination types;
- quick example.

BIO-INSPIRED ALGORITHMS

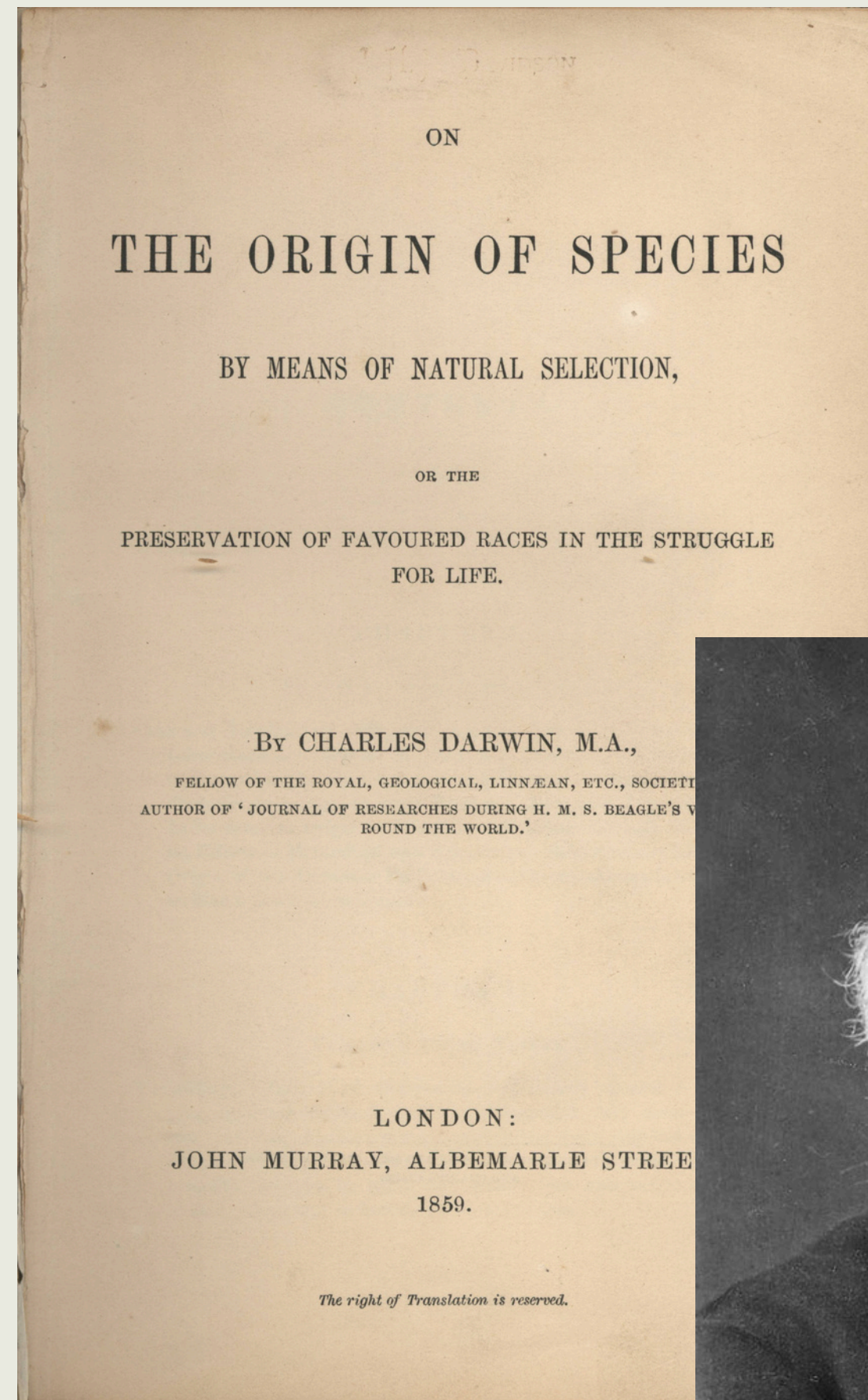


- Ant colony optimization algorithm;
- Bees algorithm;
- Bacterial colony optimization;
- evolutionary algorithms.

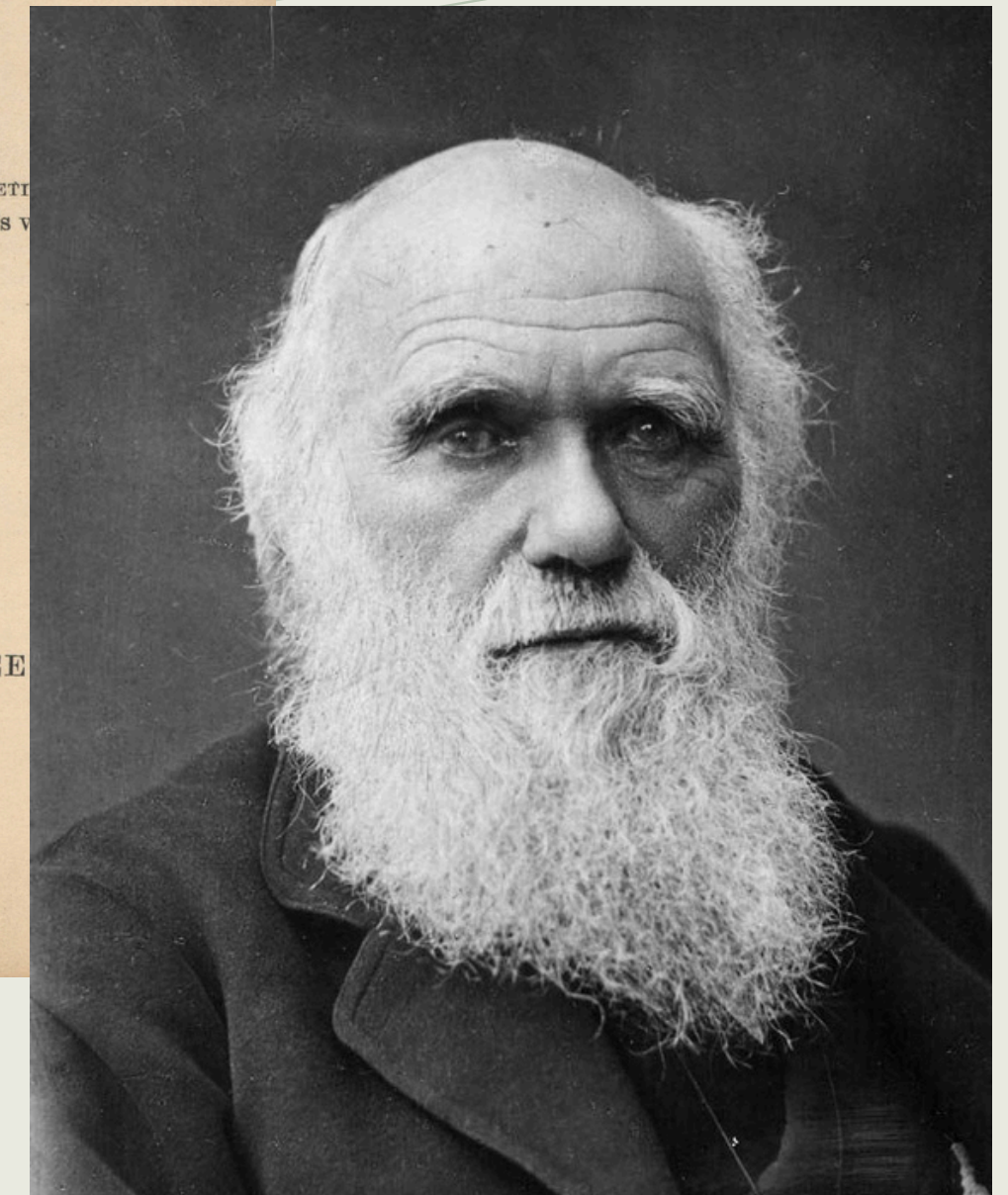


EVOLUTIONARY ALGORITHMS

- First ideas emerged in early 1950s;
- Evolutionary programming, evolution strategies, genetic algorithm

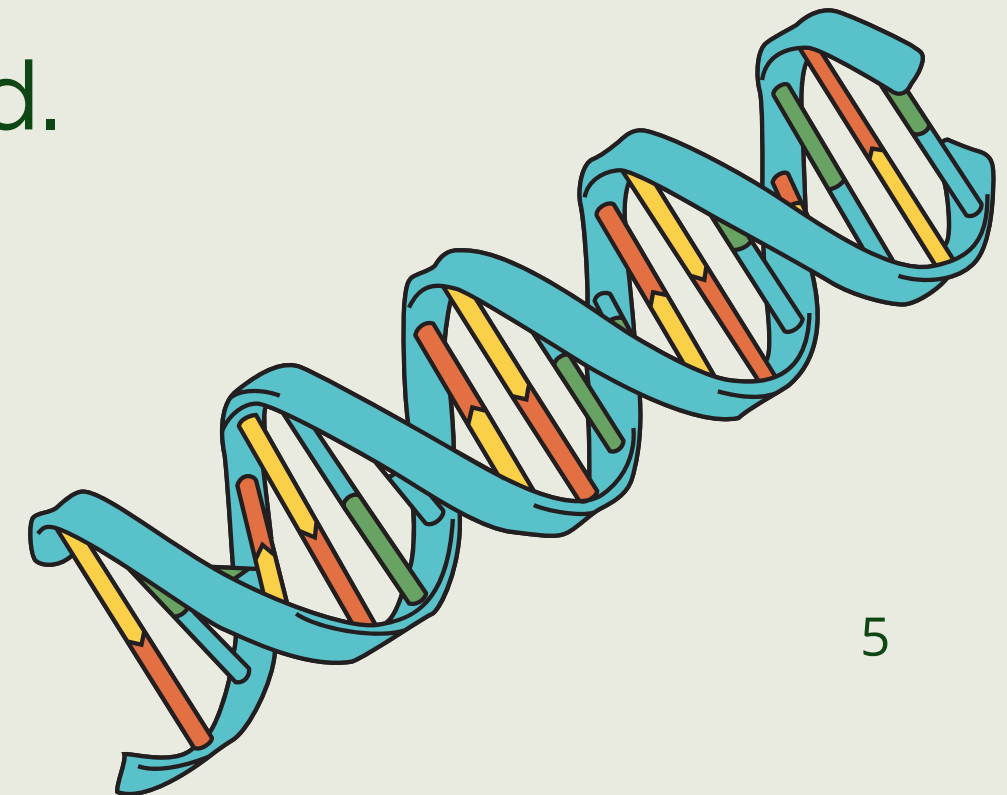


Source: Wikipedia

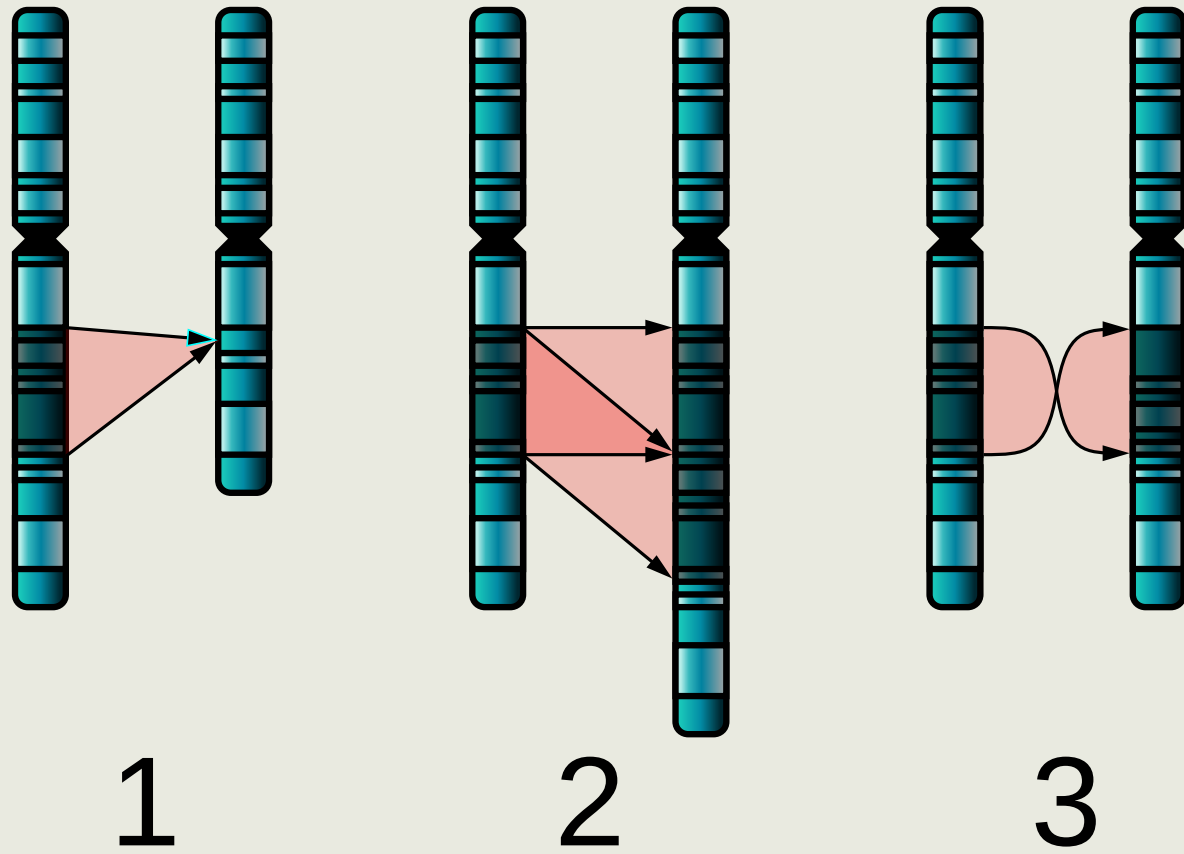


DIFFERENTIAL EVOLUTION

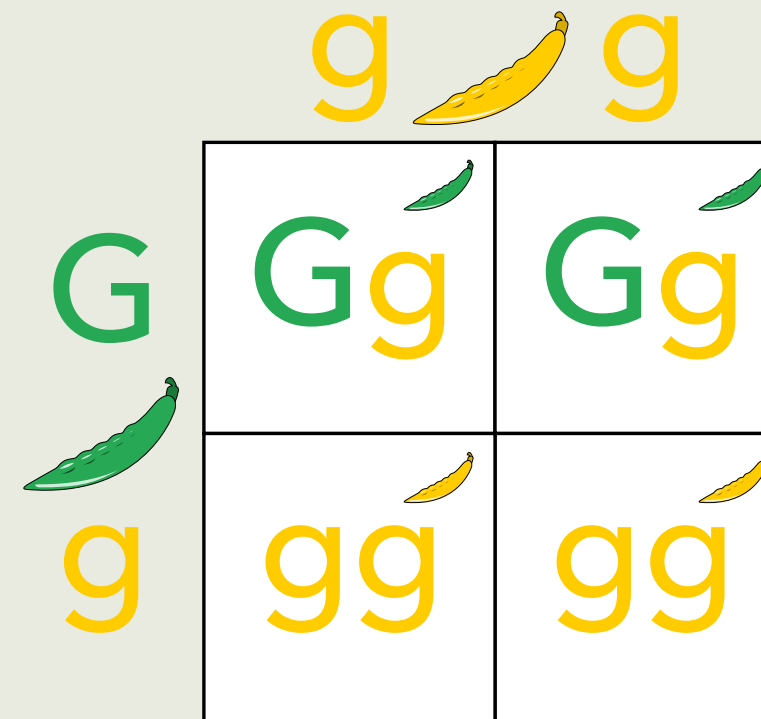
- The first written article on DE appeared as a technical report by R. Storn and K. V. Price in 1995;
- requires only few control variables;
- approach for minimizing possibly nonlinear and non differentiable continuous space functions;
- does not guarantee an optimal solution is ever found.



Mutation



Crossover



Selection

Before selection



After selection



Final population



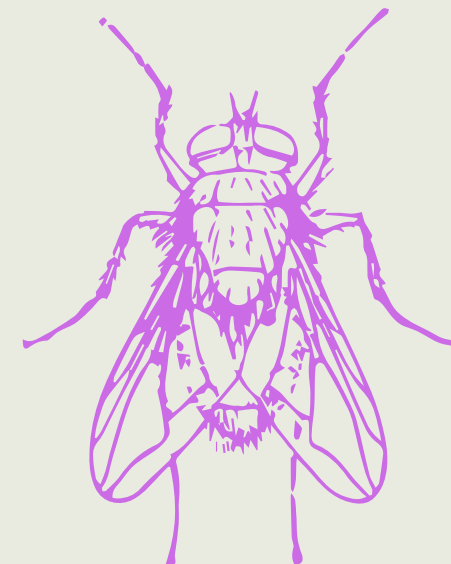
Population



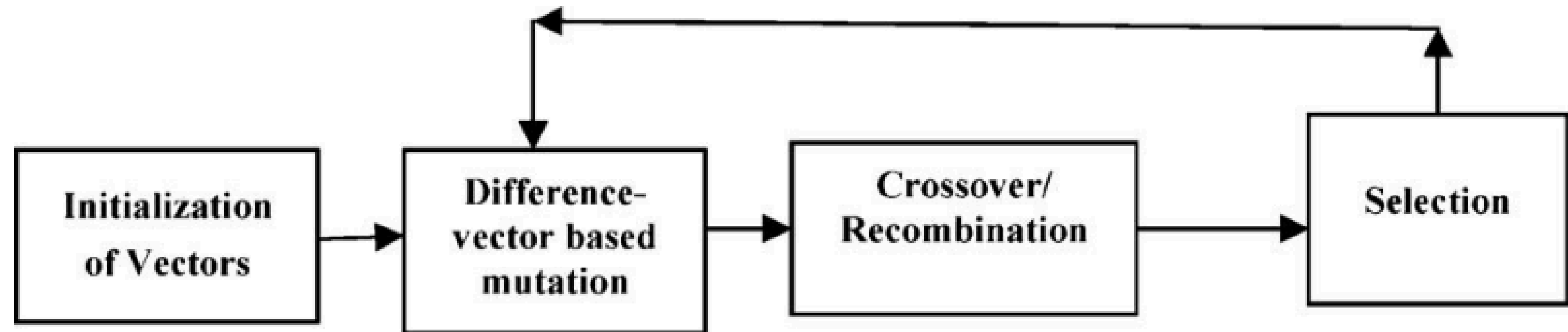
Genotype



Agent

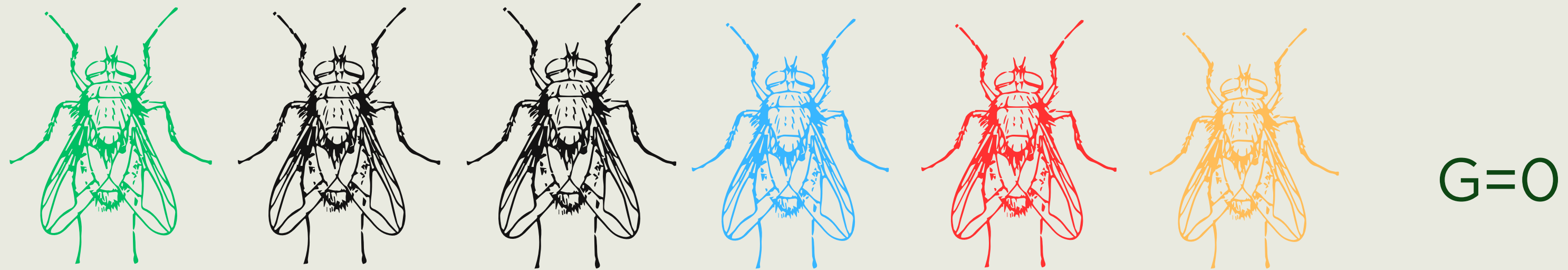


STEPS



Differential Evolution: A Survey of the State-of-the-Art, Das & Suganthan

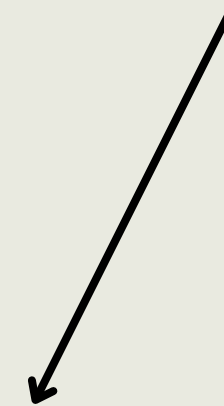
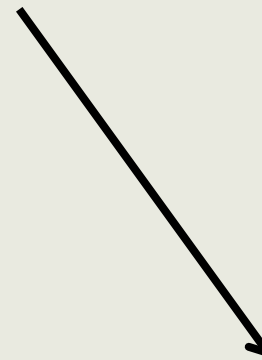
1. INITIALIZE A POPULATION



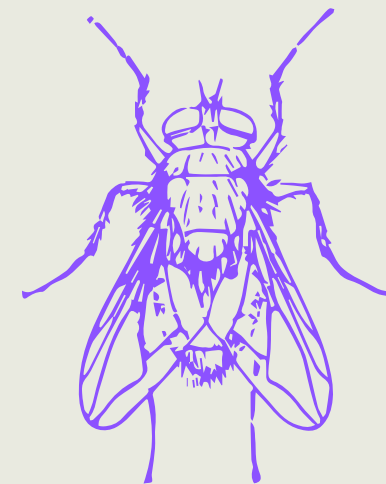
$$\vec{X}_{i,G} = [x_{1,i,G}, x_{2,i,G}, x_{3,i,G}, \dots, x_{D,i,G}].$$

$$x_{j,i,0} = x_{j,\min} + rand_{i,j}[0, 1] \cdot (x_{j,\max} - x_{j,\min})$$

2. MUTATION



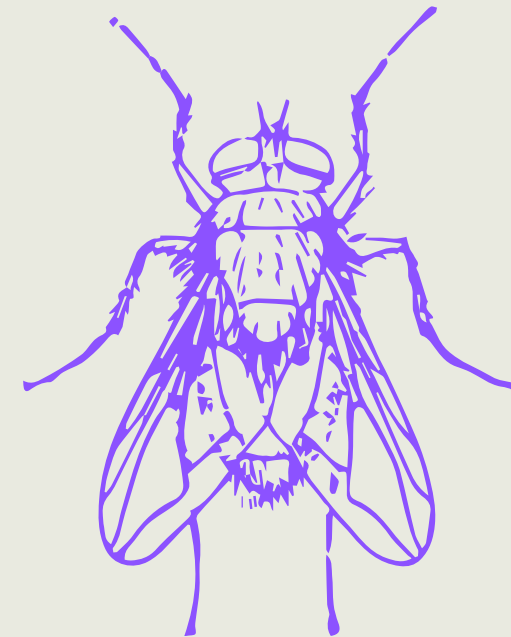
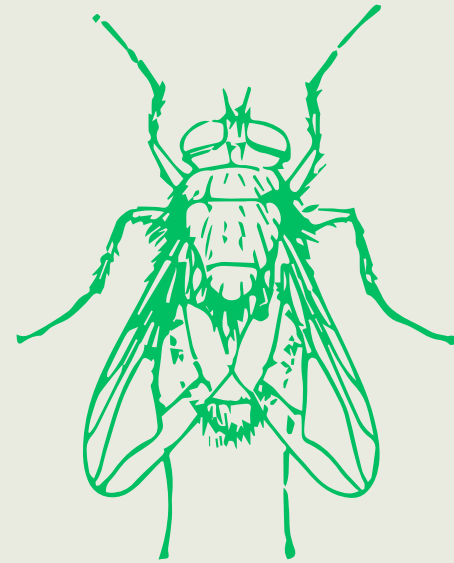
$$\vec{V}_{i,G} = \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}).$$



Donor (V)

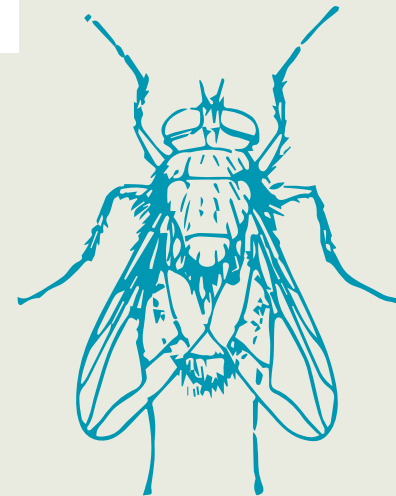
3. CROSSOVER

Target (X)



Donor (V)

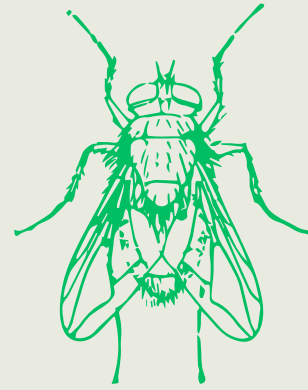
$$u_{j,i,G} = \begin{cases} v_{j,i,G} & \text{if } (rand_{i,j}[0, 1] \leq Cr \text{ or } j = j_{rand}) \\ x_{j,i,G} & \text{otherwise} \end{cases}$$



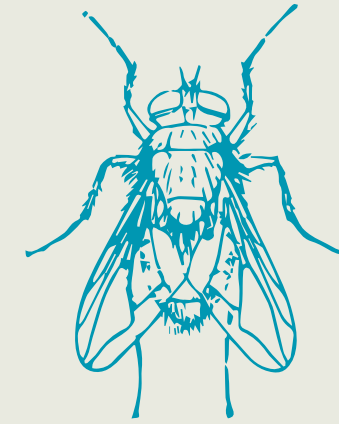
Trail (U)

4. SELECTION

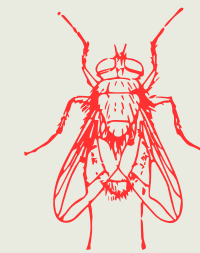
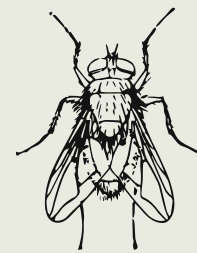
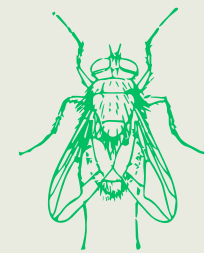
Target (X)



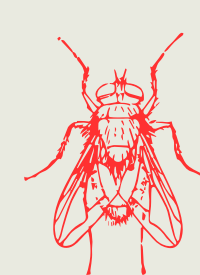
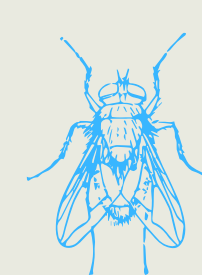
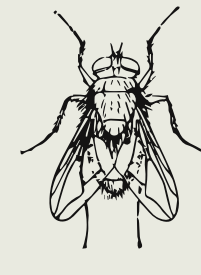
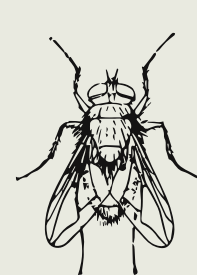
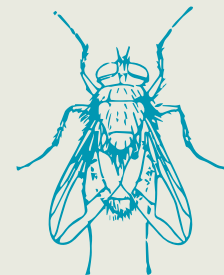
Trail (U)



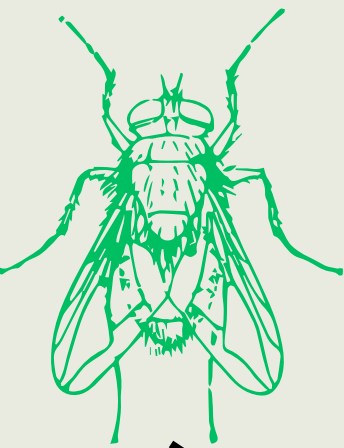
$$\begin{aligned}\vec{X}_{i,G+1} &= \vec{U}_{i,G} && \text{if } f(\vec{U}_{i,G}) \leq f(\vec{X}_{i,G}) \\ &= \vec{X}_{i,G} && \text{if } f(\vec{U}_{i,G}) > f(\vec{X}_{i,G})\end{aligned}$$



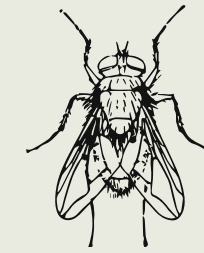
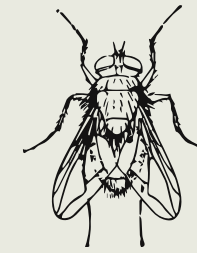
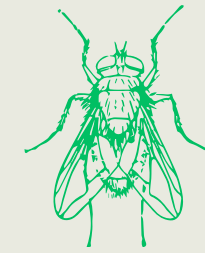
or



G=1

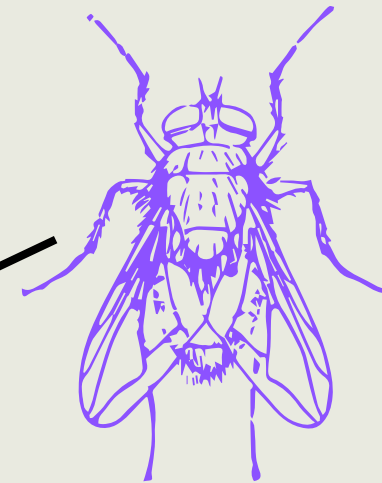


Target

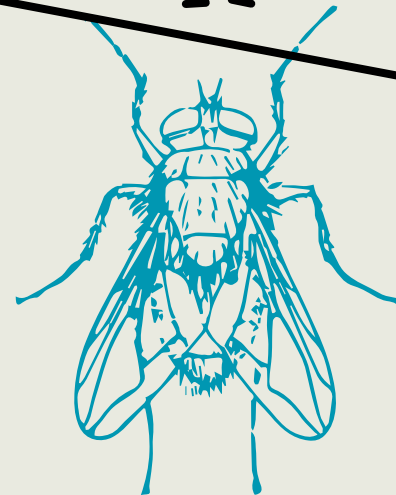


Mutation

Crossover

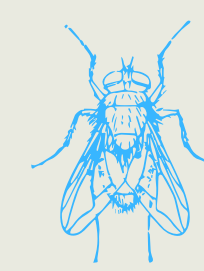
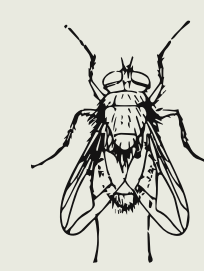
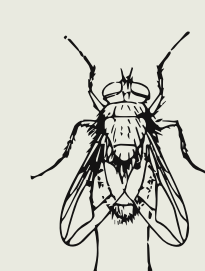
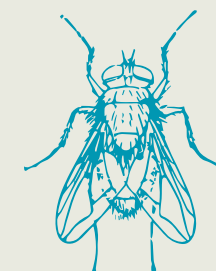


Donor



Trail

Selection



PARAMETER TUNING

- F - scale factor, usually between 0.4 and 1;
- CR - crossover rate between 0 and 1;
- NP- population size 5D - 10D.

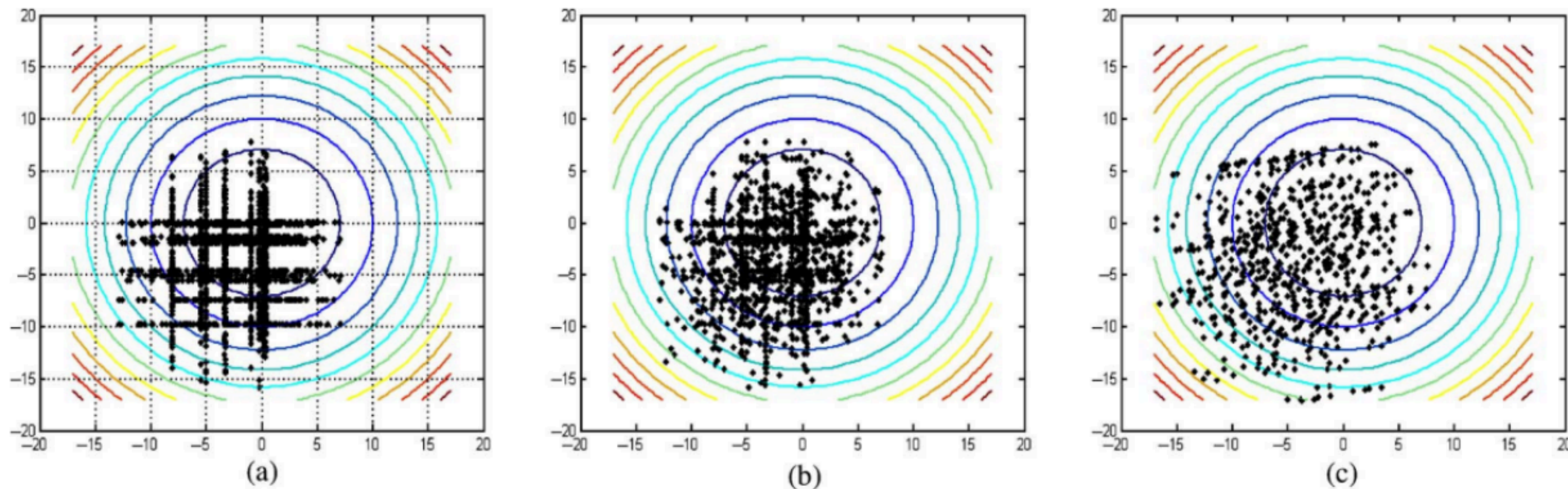
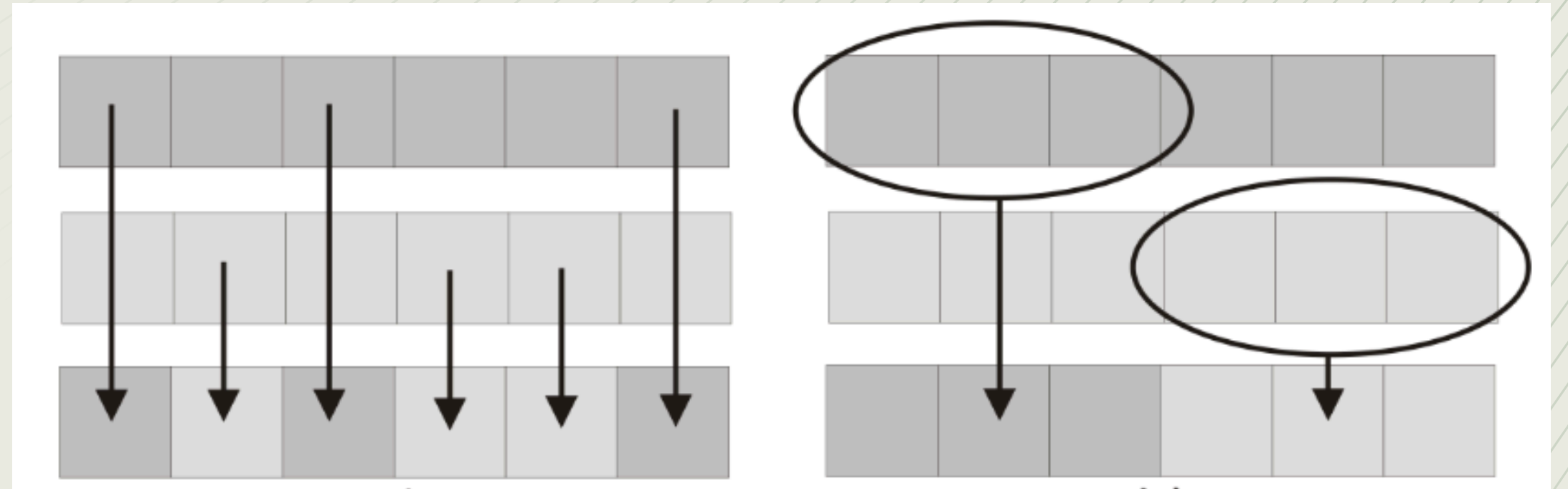


Fig. 4. Empirical distributions of candidate trial vectors for three different Cr values. (a) $Cr = 0$. (b) $Cr = 0.5$. (c) $Cr = 1.0$.

CROSSOVER



Binomial

Exponential

$$\begin{aligned} u_{j,i,G} &= v_{j,i,G} \quad \text{for } j = \langle n \rangle_D, \langle n + 1 \rangle_D, \dots, \langle n + L - 1 \rangle_D \\ x_{j,i,G} &\text{ for all other } j \in [1, D] \end{aligned} \quad (4)$$

where the angular brackets $\langle \rangle_D$ denote a modulo function with modulus D . The integer L is drawn from $[1, D]$ according to the following pseudo-code:

```

L = 0; DO
{
  L = L + 1;
} WHILE ((rand(0, 1) ≤ Cr) AND (L ≤ D)).
    
```

MUTATION

- 5 types

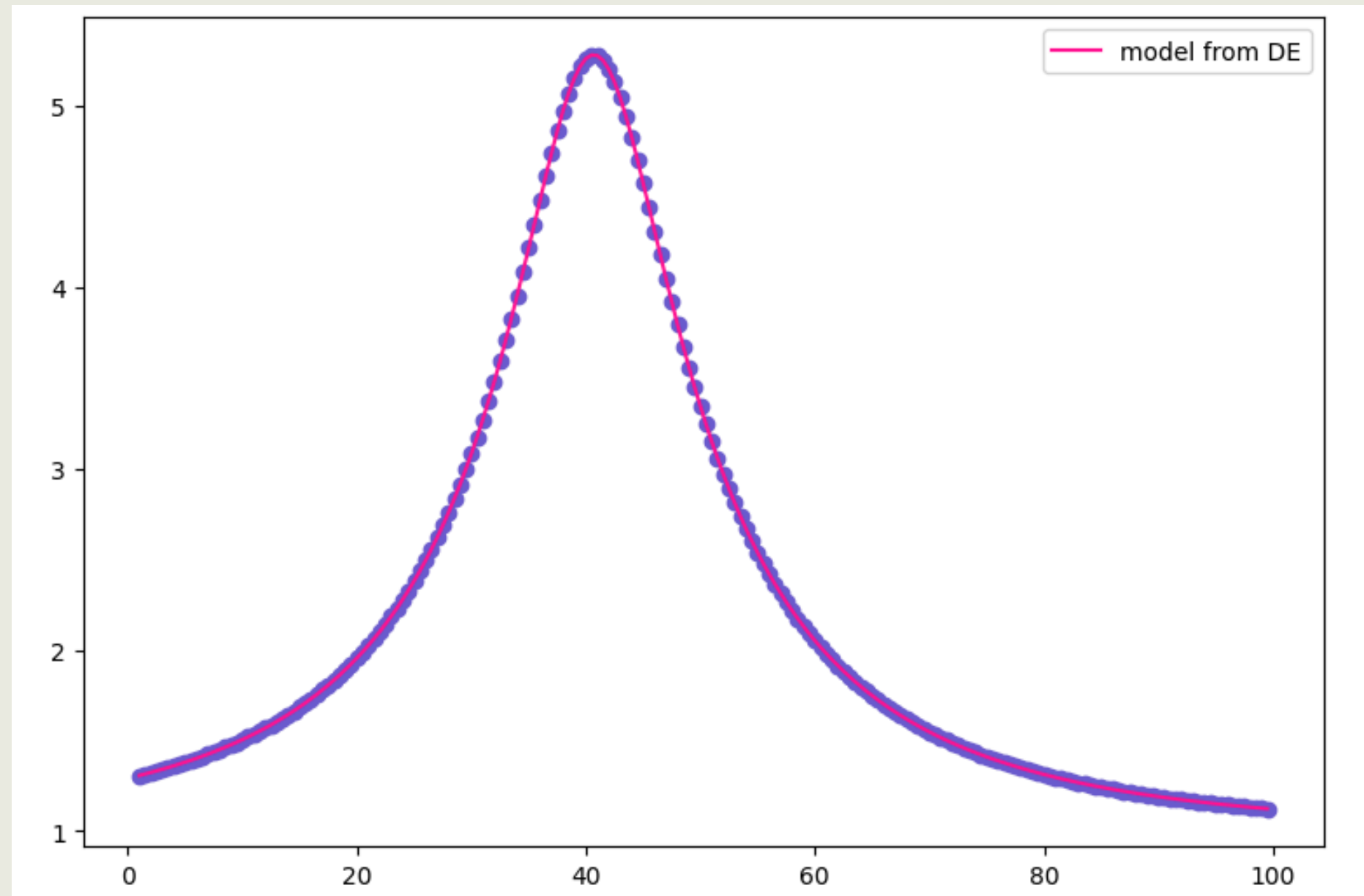
$$\text{"DE/best/1 :"} \vec{V}_{i,G} = \vec{X}_{best,G} + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}) \quad (7)$$

$$\begin{aligned} \text{"DE/target - to - best/1 :"} \vec{V}_{i,G} = & \vec{X}_{i,G} \\ & + F \cdot (\vec{X}_{best,G} - \vec{X}_{i,G}) + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}) \end{aligned} \quad (8)$$

$$\begin{aligned} \text{"DE/best/2 :"} \vec{V}_{i,G} = & \vec{X}_{best,G} + F \cdot (\vec{X}_{r_1^i,G} - \vec{X}_{r_2^i,G}) \\ & + F \cdot (\vec{X}_{r_3^i,G} - \vec{X}_{r_4^i,G}) \end{aligned} \quad (9)$$

$$\begin{aligned} \text{"DE/rand/2 :"} \vec{V}_{i,G} = & \vec{X}_{r_1^i,G} + F \cdot (\vec{X}_{r_2^i,G} - \vec{X}_{r_3^i,G}) \\ & + F \cdot (\vec{X}_{r_4^i,G} - \vec{X}_{r_5^i,G}). \end{aligned} \quad (10)$$

IMPLEMENTATION WITH SCIPY



Bounds:

- t_0 : [1,100]
- u_0 : [0,1]
- t_E : [0,100]

message: Optimization terminated successfully.

success: True

fun: 0.0

x: [4.068e+01 1.918e-01 3.834e+01]
[40.678 0.19184 38.34332]

nit: 288

nfev: 43398

Take home message:

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UNCONVENTIONAL PLACES