Distance errors and the stellar luminosity function

R.S. Stobie, K. Ishida, J. A. Peacock
1989, MNRAS 238, 709

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OUTLINE

- North Galactic Pole survey (Stobie and Ishida 1987)
- Stellar luminosity function
- Malmquist bias
- Lutz-Kelker correction

Luminosity function as 1989

- Poorly known for low luminosity stars at lower main sequence

Schmidt 105 cm telescope at Kiso Observatory

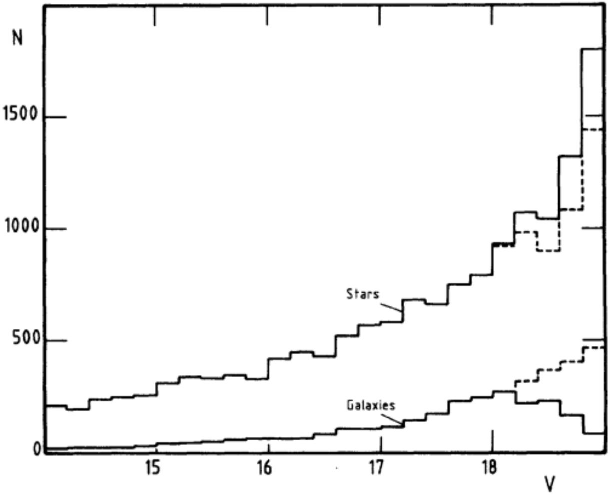
Schmidt 105 cm telescope at Kiso Observatory

- 1974 - Kiso Observatory was founded and the 105 cm Schmidt telescope was installed (105 cm - corrector plate, 150 cm - main mirror, 330 cm - focal length)
- Photographic plate have been used for almost 20 years.
- 1993 - first CCD camera (1kx1k) was installed.
- 2014 - instalation of optical wide-field video observation system:
  - 84 CMOS chips (190 Mpix),
  - field of view 20 deg^2,
  - 2 frames per second, 30 TB/night.
  - limiting magnitude - 17
  - 12 000 deg^2 in 2.5 hours

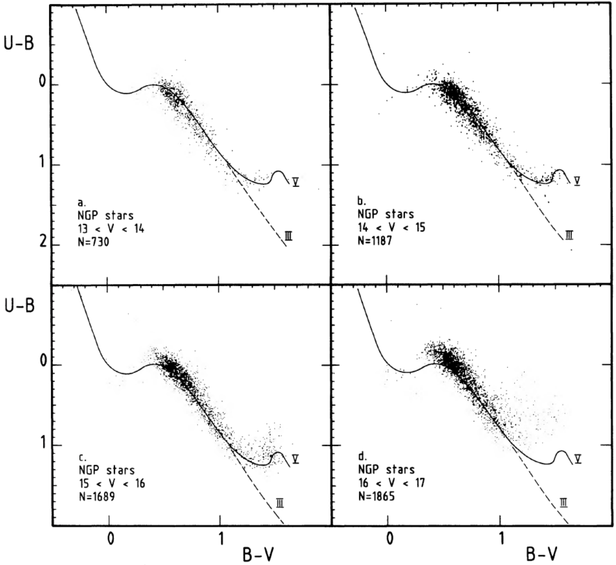
North Pole survey (Stobie and Ishida 1987)

- Observations covered 21.46 square deg, centered at R.A $=$ $13^h\ 00^m$, Dec $= 30^o\ 00'$ (l $= 80.8^o$, b $= 86.5^o$),
- 19 photograpic plates:
  - 6 U, exp time 27–70 min, limit 19.6 mag
  - 5 B, exp time 20–40 min, limit 20.4 mag
  - 5 V, exp time 20–40 min, limit 19.6 mag
  - 3 I, exp time 30–45 min limit 17.5 mag
- 356 mm square plate, 62.58"/mm
- resulting catalog 18303 stars with $V < 19$ and $B < 20$

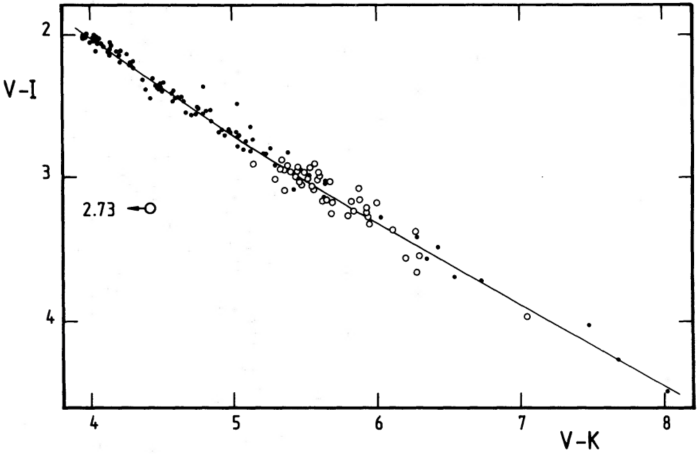
Source counts (Stobie and Ishida 1987)



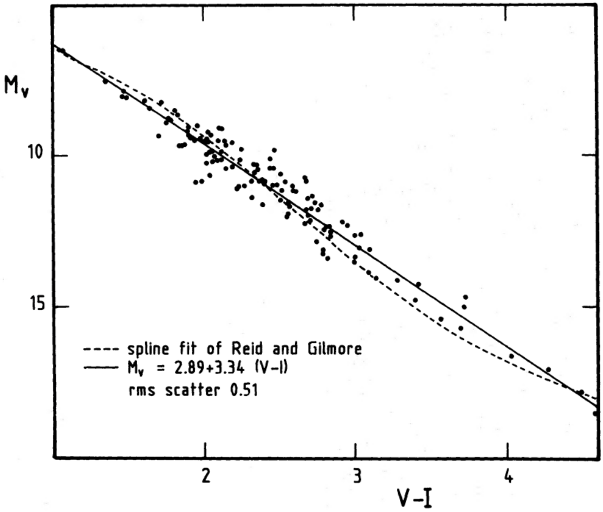
Two color diagram (Stobie and Ishida 1987)



Low contaminaion of giant stars mased on JHK photometry of Legget and Hawkins (1988)



Absolute magnitude calibration



Volume limited sample

- selection of stars with $I \leq 16$, $V > 13$, $V - I > 1.5$
- $M_V = 2.89 + 3.34(V - I)$
- selection of 178 stars closer than 130 pc
- Problems:
  – sample has not uniform density
  – $M_V - (V - I)$ relation has scatter - Malmquist bias

Malmquist bias

- For magnitude limited star's sample higher luminosity stars are sampled to greater distances and thus are more numerous than the lower luminosity stars.
- Absolute magnitude of magnitude limited sample $\bar{M}_m$ is smaller than absolute magnitude of volume limited sample M_o
- Systematic bias ΔM

$$\Delta M = \bar{M}_m - M_o = -\sigma^2 \frac{1}{N} \frac{dN}{dm},$$

where σ is rms scatter in absolute magnitude and $N(m)$ is differential number counts for objects of absolute magnitude M.

Generalized volume and luminosity function

- We take into account space density gradient

$$V_{gen} = \Omega \int z^2 \frac{\rho}{\rho_0} dz$$

– Ω - solid angle of the survey
– for $\rho/\rho_0 = \exp -z/h$ and $y = z/h$ we obtain:

$$V_{gen} = \Omega h^3 [2 - (y^2 + 2y + 2) \exp(-y)$$

- Unbiased maximum-likelihood estimator of luminosity function is

$$\Phi_o = \sum 1/V_a,$$

where sum is over objects in some luminosity bin and V_A is the maximum value of V_{gen} available to a given object given the sample limits.

Generalized volume and luminosity function

- Variance of estimator Φ_o

$$var(\Phi_o) = \sum 1/V_a^2$$

Malmquist bias

- Differential number counts N(m)

$$N \sim y^3 \exp -y$$

$$y = z/h = h^{-1} 10^{1+0.2(m-M)}$$

$$dy/dm = 0.2 ln(10) y$$

$$\Delta M = -\sigma^2 \frac{1}{N} \frac{dN}{dm} = -0.2 \ln 10 \sigma^2 (3 - y)$$

Effect of the bias on luminosity function

$$\Delta\Phi/\Phi \approx (0.6ln10)^2\sigma^2 - 0.6ln10\sigma^2\Phi'/Phi$$

$\Phi' = d\Phi/dM$

- M^* - estimated from luminosity index (V-I), M - true value,
  – $g(m, M)$ density function on the plane $m - M$,
  – $g^*(m, M^*)$ - density function in plane $m - M^*$

$$g^*(m, M*) = \int g(m, M^* + \Delta)f(\Delta)d\Delta$$

$f(\Delta)$ - distributions of errors in M^*

$$g = 0.2ln10\Omega z^3\Phi(M) = 0.2ln10\Omega z^3\exp(-z/h)\Phi_o(M)$$

- log (g) slowly varying function of M

$$g^*/g \approx (1 - \sigma^2 G'')^{-1/2}\exp[\frac{1}{2}(G'\sigma)^2/(1 - \sigma^2 G'')]$$

Corrected luminosity function

- Generalized volume based on data from $m - M^*$ plane

$$V_a^* = \int_{m_1}^{m_2} (g^*/g) 0.2 ln 10 \Omega z^3 \exp(-z/h) dm$$

- For constant space density $g \sim 10^{-0.6M} \Phi(M)$ $g*/g$ is function of M only so gives correction

$$\Delta\Phi/\Phi \approx \frac{1}{2}\sigma^2[G'' + (G')^2] = \frac{1}{2}\sigma^2[(0.6 ln 10)^2 - 1.2 ln 10 \Phi'/\Phi + \Phi''/\Phi$$

NGP luminosity function table (integer bins)

Table 2. NGP luminosity function based on counts within 130 pc.

(a) Integral magnitude bins.

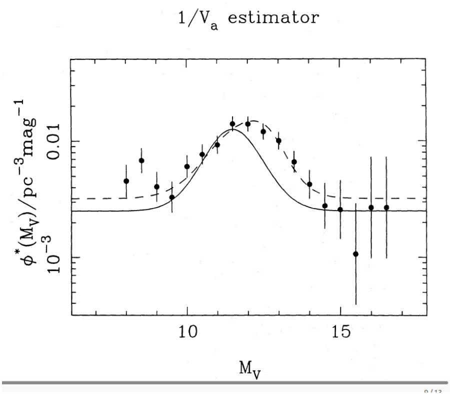
M_V (± 0.5)	N	Completeness limit (pc)	log φ^* (stars $mag^{-1} pc^{-3}$)	log φ_o (corrected)
8	6	130	-2.34	-2.42
9	15	130	-2.39	-2.48
10	16	130	-2.22	-2.26
11	33	130	-2.03	-2.01
12	51	130	-1.85	-1.95
13	37	130	-2.00	-2.25
14	16	130	-2.37	-2.53
15	3	~ 84	-2.59	-2.69
16	1	~ 61	-2.57	-2.67

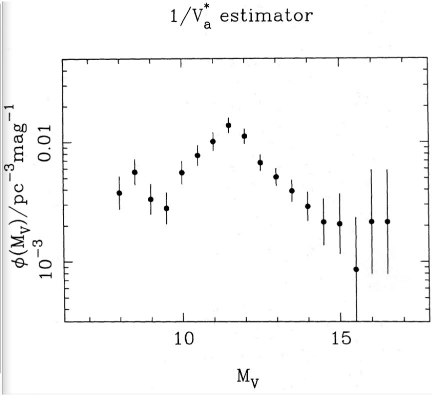
NGP luminosity function table (half integer bins)

(b) Half-integral bins.

8.5	18	130	-2.17	-2.25
9.5	7	130	-2.48	-2.55
10.5	25	130	-2.11	-2.11
11.5	45	130	-1.85	-1.86
12.5	47	130	-1.92	-2.17
13.5	27	130	-2.18	-2.41
14.5	7	~ 99	-2.56	-2.67
15.5	1	~ 72	-2.97	-3.07
16.5	1	~ 52	-2.57	-2.67

NGP luminosity function uncorrected and corrected





Lutz-Kelker correction (Lutz and Kelker 1973)

We assume: – stars uniformly distributed in space

$$N(r)dr = 4pir^2dr$$

$$N(\pi)d\pi = \frac{4pid\pi}{\pi^4}$$

– distribution of observed parallax π_o about true paralax π

$$g(\pi_o|\pi) = \frac{1}{\sqrt{(2pi)\sigma}} exp\left(-\frac{(\pi_o - \pi)^2}{2\sigma^2}\right)$$

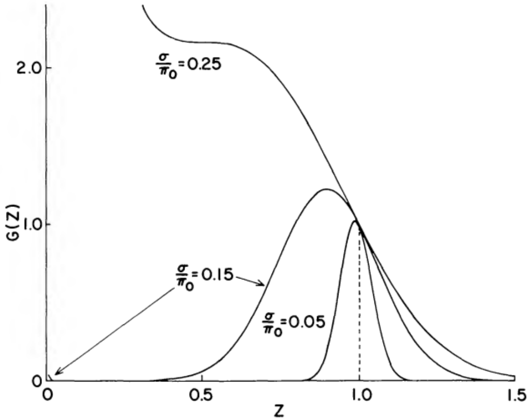
– distribution of true parallax π about observed parallax π_o

$$g(\pi|\pi_o) = \frac{g(\pi_o|\pi)g(\pi)}{g(\pi_o)} \sim \frac{1}{\pi^4} exp\left(-\frac{(\pi - \pi_o)^2}{2\sigma^2}\right) \sim \frac{\pi_o^4}{\pi^4} exp\left(-\frac{(\pi - \pi_o)^2}{2\sigma^2}\right.$$

Lutz-Kelker correction

Dimensionless parameter $Z = \frac{\pi}{\pi_o}$

$$g(\pi|\pi_o) \sim \frac{\pi_o^4}{\pi^4} exp\left(-\frac{(\pi-\pi_o)^2}{2\sigma^2}\right) \sim G(Z) = \frac{1}{Z^4} exp\left(-\frac{(Z-1)^2}{2(\sigma/\pi_o)^2}\right)$$



Lutz - Kelker correction - absolute magnitudes

$$\Delta M = M_t - M_o = 5 \log \frac{\pi}{pi_o} = 5 \log Z$$

$$< \Delta M(\epsilon) > = \frac{5 \int_{\epsilon}^{\infty} \log Z G(Z) dZ}{\int_{\epsilon}^{\infty} G(Z) dZ}$$